Optimal Composition of Characteristic Modes For Minimal Quality Factor Q

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Outline

1. Quality factor Q
2. Minimization of quality factor Q
3. Results: Quality factor Q
4. Results: Sub-optimality of G/Q
5. Excitation of optimal currents
6. Conclusion

In this talk:

- electric currents in vacuum,
- only surface regions are treated,
- all quantities in their matrix form, i.e. operators $\to$ matrices, functions $\to$ vectors,
- small electrical size is considered, i.e. $ka < 1$,
- time-harmonic quantities, i.e., $\boldsymbol{\mathcal{A}}(\boldsymbol{r}, t) = \sqrt{2}\,\mathrm{Re}\left\{\boldsymbol{A}(\boldsymbol{r})\exp(\mathrm{j}\omega t)\right\}$ are considered.

Minimization of quality factor Q

Quality factor Q ...

- is (generally) proportional to FBW,
- therefore, of interest for ESA ($ka < 1$).

Fundamental bounds of quality factor Q

- are known for several canonical bodies,
- many interesting works recently appeared[1],
  - still, they are unknown for arbitrarily shaped bodies.

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[1] M. Gustafsson, C. Sohl, and G. Kristensson, "Physical limitations on antennas of arbitrary shape", *Proc. of Royal Soc. A*, vol. 463, pp. 2589–2607, 2007. DOI: 10.1098/rspa.2007.1893
M. Gustafsson, D. Tayli, C. Ehrenborg, *et al.*, "Tutorial on antenna current optimization using MATLAB and CVX", , *FERMAT*, 2015
O. S. Kim, "Lower bounds on Q for finite size antennas of arbitrary shape", *IEEE Trans. Antennas Propag.*, vol. 64, no. 1, pp. 146–154, 2016. DOI: 10.1109/TAP.2015.2499764

Minimization of quality factor Q

Current $\mathbf{I}_{\mathrm{opt}}$ minimizing quality factor Q of a given shape Ω:

$$Q\left(\mathbf{I}_{\mathrm{opt}}\right) = \min_{\mathbf{I}} \{ Q\left(\mathbf{I}\right) \} \tag{1}$$

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Procedure followed in this talk[2]:

[2] M. Capek and L. Jelinek, "Optimal composition of modal currents for minimal quality factor Q", , 2016, arXiv:1602.04808

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Step 1+2: Definition of Q and $\widetilde{W}_{\mathrm{sto}}$

Quality factor Q defined by parts as

$$Q(\mathbf{I}) = Q_{\mathrm{U}}(\mathbf{I}) + Q_{\mathrm{ext}}(\mathbf{I}) \tag{2}$$

using stored energy[3]

$$Q_{\mathrm{U}}(\mathbf{I}) = \frac{\omega \widetilde{W}_{\mathrm{sto}}}{P_{\mathrm{r}}} = \frac{\mathbf{I}^{\mathrm{H}}\mathbf{X}'\mathbf{I}}{2\mathbf{I}^{\mathrm{H}}\mathbf{R}\mathbf{I}} = \frac{\mathbf{I}^{\mathrm{H}}\omega \frac{\partial \mathbf{X}}{\partial \omega}\mathbf{I}}{2\mathbf{I}^{\mathrm{H}}\mathbf{R}\mathbf{I}}, \tag{3}$$

and tuning

$$Q_{\mathrm{ext}}(\mathbf{I}) = \frac{\left|\mathbf{I}^{\mathrm{H}}\mathbf{X}\mathbf{I}\right|}{2\mathbf{I}^{\mathrm{H}}\mathbf{R}\mathbf{I}}. \tag{4}$$

$$\boldsymbol{J} \approx \sum_n I_n \boldsymbol{f}_n, \quad \mathbf{Z} = \mathbf{R} + \mathrm{j}\mathbf{X}$$

[3] M. Cismasu and M. Gustafsson, "Antenna bandwidth optimization with single freuquency simulation", *IEEE Trans. Antennas Propag.*, vol. 62, no. 3, pp. 1304–1311, 2014, R. F. Harrington and J. R. Mautz, "Control of radar scattering by reactive loading", *IEEE Trans. Antennas Propag.*, vol. 20, no. 4, pp. 446–454, 1972. DOI: 10.1109/TAP.1972.1140234, G. A. E. Vandenbosch, "Reactive energies, impedance, and Q factor of radiating structures", *IEEE Trans. Antennas Propag.*, vol. 58, no. 4, pp. 1112–1127, 2010. DOI: 10.1109/TAP.2010.2041166.

Step 3: Formulation of the problem

Find $\mathbf{I}_{\mathrm{opt}}$ so that

$$\text{minimize} \quad \text{quality factor } Q, \tag{5}$$

$$\text{subject to} \quad \widetilde{W}_{\mathrm{m}} - \widetilde{W}_{\mathrm{e}} = 0. \tag{6}$$

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Searching for self-resonant current $\mathbf{I}_{\mathrm{opt}}$ fulfilling (5)–(6) is not a convex problem.

Step 4: Representation of $\mathbf{I}_{\mathrm{opt}}$

Current decomposition

Let us decompose the current into (yet unknown) modes such that

$$\mathbf{I} = \sum_{n=1}^{N} \alpha_n \mathbf{I}_n. \tag{7}$$

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Then, the quality factor Q reads

$$Q(\mathbf{I}) = \frac{\sum\limits_{v=1}^{V} \sum\limits_{u=1}^{U} \alpha_u^* \alpha_v \mathbf{I}_u^{\mathrm{H}} \mathbf{X}' \mathbf{I}_v + \left| \sum\limits_{v=1}^{V} \sum\limits_{u=1}^{U} \alpha_u^* \alpha_v \mathbf{I}_u^{\mathrm{H}} \mathbf{X} \mathbf{I}_v \right|}{2 \sum\limits_{v=1}^{V} \sum\limits_{u=1}^{U} \alpha_u^* \alpha_v \mathbf{I}_u^{\mathrm{H}} \mathbf{R} \mathbf{I}_v}. \tag{8}$$

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Analytical solution can easily be found if

$$\mathbf{I}_u^{\mathrm{H}} \mathbf{R} \mathbf{I}_v = \delta_{uv}, \tag{9}$$

$$\mathbf{I}_u^{\mathrm{H}} \mathbf{X} \mathbf{I}_v = A_{uv} \delta_{uv}, \tag{10}$$

$$\mathbf{I}_u^{\mathrm{H}} \mathbf{X}' \mathbf{I}_v = B_{uv} \delta_{uv}. \tag{11}$$

Step 4: Representation of $\mathbf{I}_{\mathrm{opt}}$

Optimal current

Normalizing $\alpha_1 = 1$, we get the result[4] if

- tuning is represented by localized current (i.e. external tuning element) as

$$Q\left(\mathbf{I}_{\mathrm{opt}}\right) = \frac{\mathbf{I}_1^{\mathrm{H}}\mathbf{X}'\mathbf{I}_1 + \left|\mathbf{I}_1^{\mathrm{H}}\mathbf{X}\mathbf{I}_1\right|}{2}, \tag{12}$$

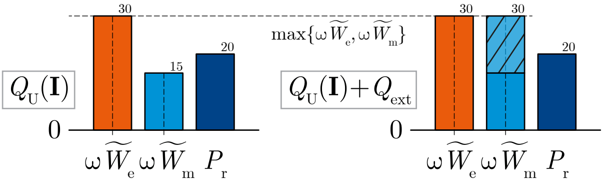
- tuning is represented by low-order modal current as

$$Q\left(\mathbf{I}_{\mathrm{opt}}\right) = \frac{\mathbf{I}_1^{\mathrm{H}}\mathbf{X}'\mathbf{I}_1 + \left|\alpha_{\mathrm{opt}}\right|^2 \mathbf{I}_2^{\mathrm{H}}\mathbf{X}'\mathbf{I}_2}{2\left(1 + \left|\alpha_{\mathrm{opt}}\right|^2\right)}. \tag{13}$$

Both options are discussed in the following figure...

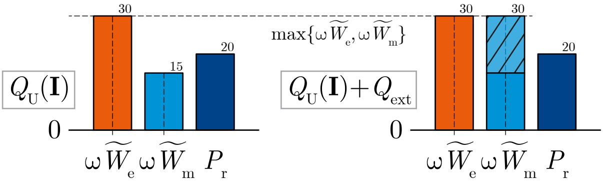
[4] M. Capek and L. Jelinek, “Optimal composition of modal currents for minimal quality factor Q”, , 2016, arXiv:1602.04808

Localized and distributive tunning



Tuning by external lumped element (localized current).

Localized and distributive tunning



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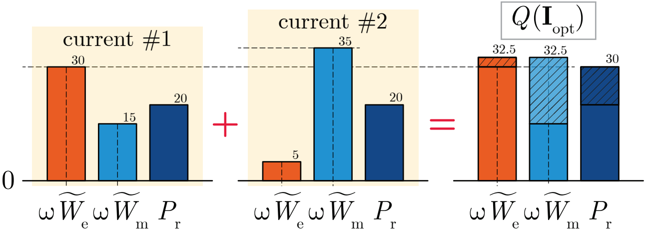

Tuning by distributive current.

Step 5: Optimal composition to form $\mathbf{I}_{\mathrm{opt}}$

To diagonalize $\mathbf{R}$, $\mathbf{X}$ and $\mathbf{X}'$ we can choose:

$$\mathbf{X}\mathbf{I}_u = \lambda_u \mathbf{R}\mathbf{I}_u, \tag{16}$$

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$$\mathbf{X}\mathbf{I}_u = \lambda_u \mathbf{R}\mathbf{I}_u, \tag{14}$$

$$\mathbf{X}'\mathbf{I}_u = \xi_u \mathbf{R}\mathbf{I}_u, \tag{15}$$

$$\mathbf{X}\mathbf{I}_u = \chi_u \mathbf{X}'\mathbf{I}_u. \tag{16}$$

- All GEPs involve only two of the three operators[5] ($\mathbf{R}$, $\mathbf{X}$, $\mathbf{X}'$).

[5] Modal currents have cross-terms with the non-diagonalized operator, e.g., for (14) $\mathbf{I}_u^{\mathrm{H}}\mathbf{X}'\mathbf{I}_v \neq 0$.

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- All GEPs involve only two of the three operators[5] ($\mathbf{R}$, $\mathbf{X}$, $\mathbf{X}'$).
- Using characteristic modes, defined by (14), we get[6] for $\mathbf{I}_{\mathrm{opt}}$

$$\alpha_{\mathrm{opt}} = \sqrt{-\frac{\lambda_1}{\lambda_2}}\,\mathrm{e}^{\mathrm{j}\varphi}, \quad \varphi \in [-\pi, \pi], \quad \lambda_2 \neq 0. \tag{17}$$

[5] Modal currents have cross-terms with the non-diagonalized operator, e.g., for (14) $\mathbf{I}_u^{\mathrm{H}}\mathbf{X}'\mathbf{I}_v \neq 0$.

[6] M. Capek and L. Jelinek, "Optimal composition of modal currents for minimal quality factor Q", , 2016, arXiv:1602.04808

A spherical shell

Minimization of quality factor Q

- Special case for which $\mathbf{R}$, $\mathbf{X}$ and $\mathbf{X}'$ are all diagonalizable.

Optimal ratio between dominant (TM) and tuning (TE) modes:

$$\alpha_{\mathrm{opt}} = \sqrt{-\frac{\lambda_{\mathrm{TM10}}}{\lambda_{\mathrm{TE10}}}}\,\mathrm{e}^{\mathrm{j}\varphi} = \sqrt{-\frac{1 - ka\dfrac{\mathrm{y}_0(ka)}{\mathrm{y}_1(ka)}}{1 - ka\dfrac{\mathrm{j}_0(ka)}{\mathrm{j}_1(ka)}}}\,\mathrm{e}^{\mathrm{j}\varphi}.$$

- arbitrary φ for minimal quality factor Q,
- specified φ for maximal G/Q (will be shown later).

A spherical shell

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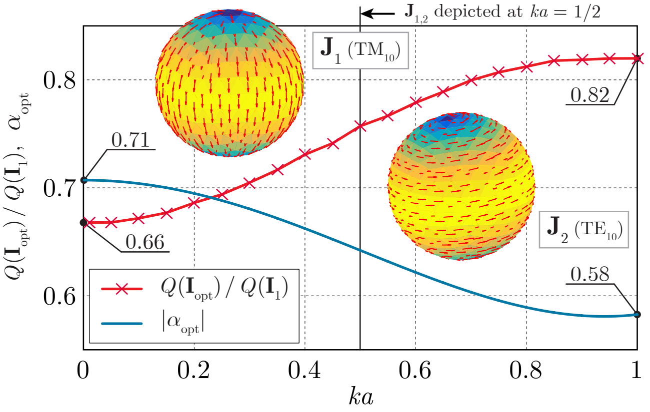
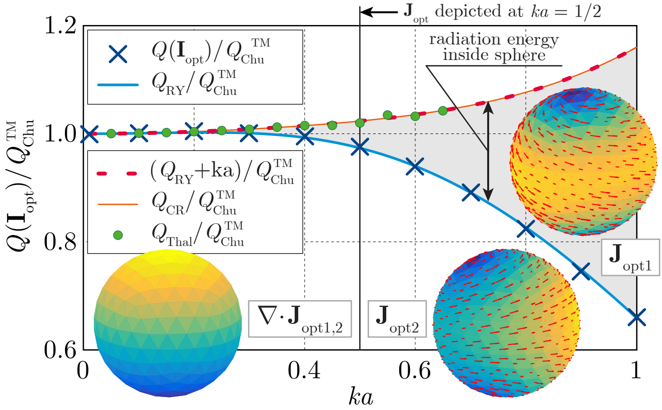


Normalized quality factor Q and reduction rate α_{opt} for a spherical shell.

A spherical shell

Comparison with fundamental bounds

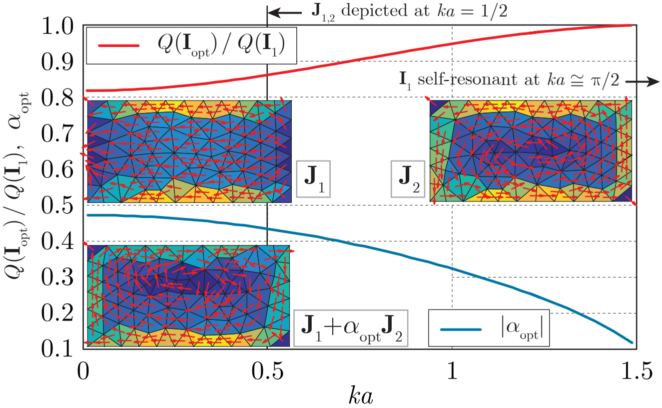


Comparison of various[7] "minimal" quality factors Q of a spherical shell normalized to $Q_{\mathrm{Chu}}^{\mathrm{TM}}$.

[7] Q_{RY} – Rhodes (1976), Yaghjian and Best (2005), Vandenbosch (2010), Gustafsson et al. (2013); Q_{CR} – Collin and Rothschild (1964); Q_{Thal} – Thal (2011); $Q\left(\mathbf{I}_{\mathrm{opt}}\right)$ – this work.

A rectangular plate

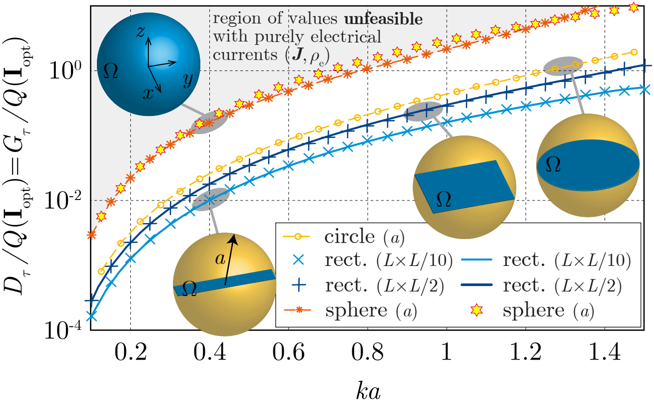
- The cross-terms $\mathbf{I}_u^{\mathrm{H}} \mathbf{X}' \mathbf{I}_v$ are negligible (for all calculated examples).



Normalized quality factor Q and reduction rate α_{opt} for $L \times L/2$ rectangular plate.

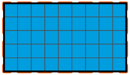
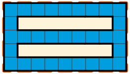
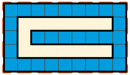
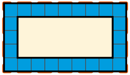
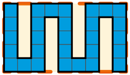
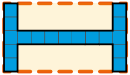
What about G/Q limits for $\mathbf{I}_{\mathrm{opt}}$?

- Current $\mathbf{I}_{\mathrm{opt}}$ found in this work yields (sub-)optimal G/Q as well.



G/Q_{opt} ratios for different canonical shapes[8].

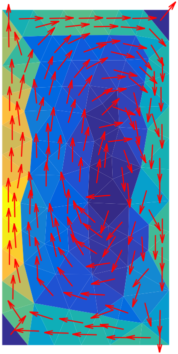
[8] Yellow asterisks – Gustafsson et al. (2007), solid blue lines – Gustafsson et al. (2015).

Ω $(ka = 0.5)$	$\frac{Q(\mathbf{I}_{\mathrm{opt}})}{Q_{\mathrm{Chu}}^{\mathrm{TM}}}$	$\frac{Q(\mathbf{I}_{\mathrm{opt}})}{Q(\mathbf{I}_1)}$	$\frac{G_y}{Q(\mathbf{I}_{\mathrm{opt}})}$	$\frac{S}{S_{\square}}$
	3.566	0.839	0.0352	1.000
	3.613	0.840	0.0349	0.689
	3.658	0.842	0.0347	0.667
	3.691	0.839	0.0343	0.533
	4.398	0.995	0.0285	0.644
	4.670	1.000	0.0283	0.378

[9] G. A. E. Vandenbosch, "Explicit relation between volume and lower bound for Q for small dipole topologies", *IEEE Trans. Antennas Propag.*, vol. 60, no. 2, pp. 1147–1152, 2012. DOI: 10.1109/TAP.2011.2173127

Optimal currents × optimal antennas

$$Q\left(\mathbf{I}_{\mathrm{opt}}\right)/Q_{\mathrm{Chu}}^{\mathrm{TM}} = 4.85$$



Optimal current $\mathbf{I}_{\mathrm{opt}}$.

[10] S. R. Best, “Electrically small resonant planar antennas”, *IEEE Antennas Propag. Magazine*, vol. 57, no. 3, pp. 38–47, 2015. DOI: 10.1109/MAP.2015.2437271

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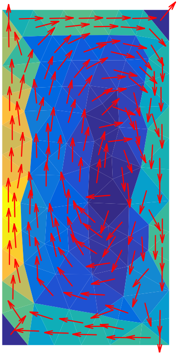
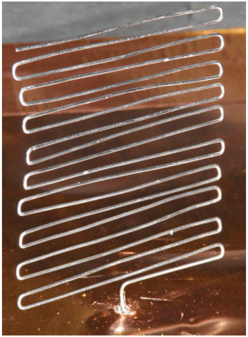
$Q/Q_{\mathrm{Chu}}^{\mathrm{TM}} = 6.05$



Optimal current $\mathbf{I}_{\mathrm{opt}}$.

same ka



Near-optimal antenna[10].

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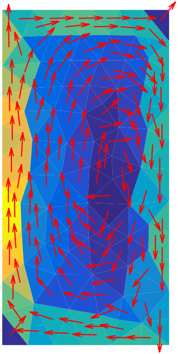
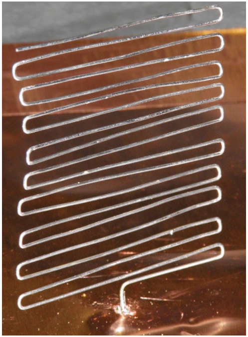


Optimal current $\mathbf{I}_{\mathrm{opt}}$.

☹ FEEDING ✓



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Excitation: NP-hard problem?

Finding the current $\mathbf{I}_{\mathrm{opt}}$ is only a (small) part of a synthesis since it is incompatible with any realistic feeding.

- Proper feeding position(s) must be determined.
- Shape Ω must be modified.

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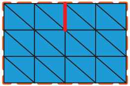
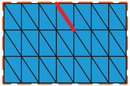
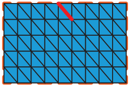
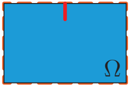
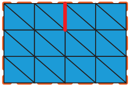
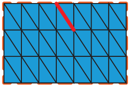
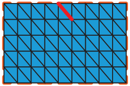
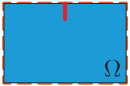
How much DOF we have?				
N (unknowns)	28	52	120	∞
possibilities				
unique solutions				

Complexity of geometrical optimization for given voltage gap (red line) and N unknowns.

Excitation: NP-hard problem?

Finding the current $\mathbf{I}_{\mathrm{opt}}$ is only a (small) part of a synthesis since it is incompatible with any realistic feeding.

- Proper feeding position(s) must be determined.
- Shape Ω must be modified.

How much DOF we have?				
N (unknowns)	28	52	120	∞
possibilities	$5.24 \cdot 10^{29}$	$1.39 \cdot 10^{68}$	$1.15 \cdot 10^{199}$	∞
unique solutions	$2.68 \cdot 10^{8}$	$4.50 \cdot 10^{15}$	$1.33 \cdot 10^{36}$	∞

Complexity of geometrical optimization for given voltage gap (red line) and N unknowns.

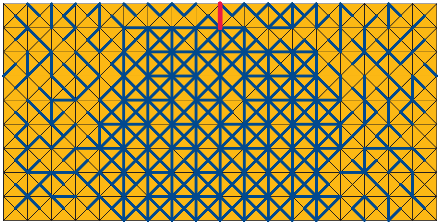
Antenna synthesis – how far can we go?

- On the present, only the heuristic optimization. . .

Excitation: What is $\mathbf{I}_{\text{opt}}$ good for?

Excitation placement is *ad hoc.*

Computational time: 12116 s



Result of heuristic structural optimization using MOGA NSGAII (Q_{ext}, Q_{U}) from AToM-FOPS.

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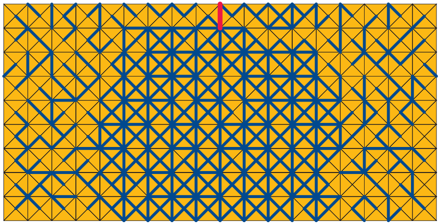
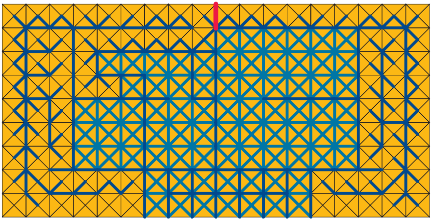
Computational time: 12116 s



Result of heuristic structural optimization using MOGA NSGAII (Q_{ext}, Q_{U}) from AToM-FOPS.

Computational time: 1155 s

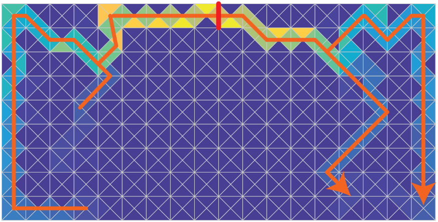


Result of deterministic in-house algorithm removing in each iteration the "worse" edge.

Excitation: What is $\mathbf{I}_{\mathrm{opt}}$ good for?

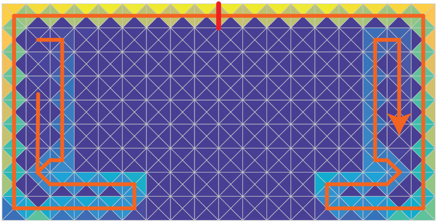
Excitation placement is *ad hoc*.

$$Q\left(\mathbf{I}\right)/Q_{\mathrm{Chu}}^{\mathrm{TM}} = 7.23$$



Resulting sub-optimal current approaching minimal value of quality factor Q.

$$Q\left(\mathbf{I}\right)/Q_{\mathrm{Chu}}^{\mathrm{TM}} = 7.24$$

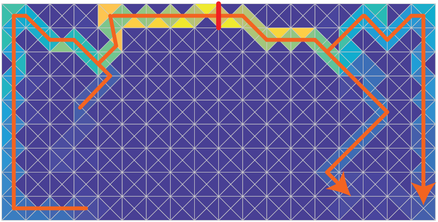


Resulting current given by in-house deterministic algorithm.

Excitation: What is $\mathbf{I}_{\mathrm{opt}}$ good for?

Excitation placement is *ad hoc*.

$$Q(\mathbf{I})/Q_{\mathrm{Chu}}^{\mathrm{TM}} = 7.23$$



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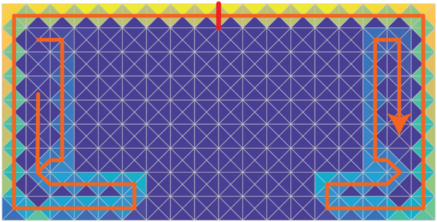

Resulting current given by in-house deterministic algorithm.

Depicted currents $\mathbf{I}$ are completely different from $\mathbf{I}_{\mathrm{opt}}$!

- Optimal currents are incompatible with realistic (fed) scenarios.

Conclusion

Optimal current $\mathbf{I}_{\mathrm{opt}}$ approaching lower bounds of quality factor Q can easily be obtained assuming:

- small ka (negligible cross-terms),
- electrical currents,
- surface geometries.

(Sub-)optimal currents for G, G/Q, η_{rad} etc. can be found if proper GEP (modal decomposition) is utilized.

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Similar work of the same topic recently published[11].

Talk relevant to this presentation:

- L. Jelinek and M. Capek: Optimal Currents in the Characteristic Modes Basis[12], session MO–A1.4P, **Mo (14:20)**.

[11] J. Chalas, K. Sertel, and J. L. Volakis, "Computation of the Q limits for arbitrary-shaped antennas using characteristic modes", *IEEE Trans. Antennas Propag. (Early Access)*, vol. PP, pp. 1–11, 2016. DOI: 10.1109/tap.2016.2557844

[12] L. Jelinek and M. Capek, "Optimal currents on arbitrarily shaped surfaces", , 2016, arXiv:1602.05520

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Future work

- Excitation placement, number of feeders.
- Shape modifications.
- Deeper understanding of the relationship between optimal currents and optimal antennas.

Questions?

For complete PDF presentation see ‣ capek.elmag.org

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