Recent Development in the Stored Energy Evaluation

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Outline

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  - since

$$\boldsymbol{E}_{\mathrm{far}} \left(\vartheta, \varphi\right) \approx \frac{1}{r} \boldsymbol{F} \left(\vartheta, \varphi\right) \mathrm{e}^{-\mathrm{j}kr},$$

$$\boldsymbol{H}_{\mathrm{far}} \left(\vartheta, \varphi\right) \approx \sqrt{\frac{\epsilon_0}{\mu_0}} \mathbf{n}_0 \times \boldsymbol{E}_{\mathrm{far}} \left(\vartheta, \varphi\right).$$

Stored energy in EM field

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- Stored energy W_{sto} is important for evaluation of antenna quality factor Q, since[1]:

The ratio of 2π times the energy stored in the fields excited by the antenna to the energy radiated and dissipated per cycle.

*) There is also IEEE note[1] regarding the Q_Z factor. It is however not discussed here.
†) Another matter is a potential functional dependence[2] between FBW and Q.

[1] *Standard definitions of terms for antennas 145 – 1993*, IEEE Antennas and Propagation Society

[2] M. Capek, L. Jelinek, and P. Hazdra, "On the functional relation between quality factor and fractional bandwidth", *IEEE Trans. Antennas Propag.*, vol. 63, no. 6, pp. 2787–2790, 2015. DOI: 10.1109/TAP.2015.2414472.

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$$\boxed{Q \equiv \omega_0 \frac{W_{\mathrm{sto}}}{P_{\mathrm{rad}}}}$$

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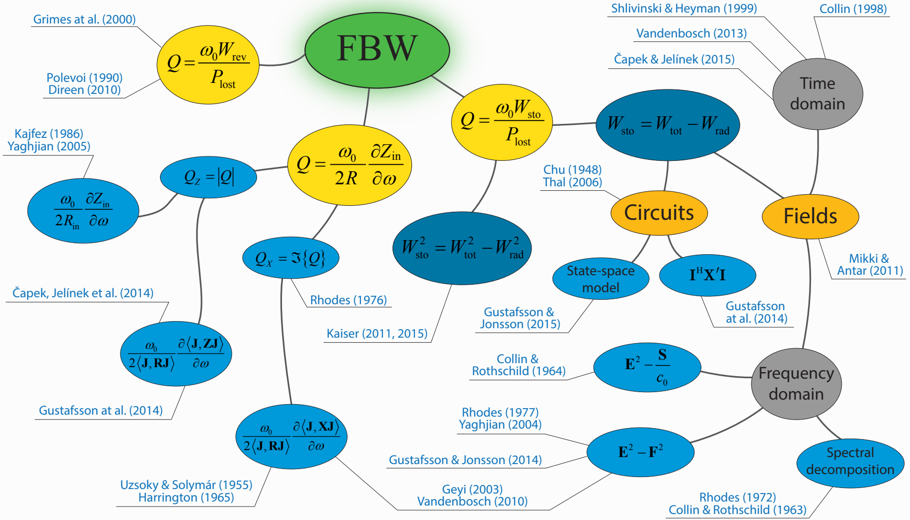

FBW is parameter of primary importance... (at least for many researchers)

What concepts we selected?

Two different points of view can be distinguished. . .

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Extraction of radiated energy[3]

$$w_{\mathrm{sto}}(\boldsymbol{r}) = w_{\mathrm{tot}}(\boldsymbol{r}) - \frac{\epsilon_0}{2} \frac{|\boldsymbol{F}(\boldsymbol{r})|^2}{r^2}$$

[3] D. R. Rhodes, "A reactance theorem", *Proc. R. Soc. Lond. A.*, vol. 353, pp. 1–10, 1977. DOI: 10.1098/rspa.1977.0018,
A. D. Yaghjian and S. R. Best, "Impedance, bandwidth and Q of antennas", *IEEE Trans. Antennas Propag.*, vol. 53, no. 4, pp. 1298–1324, 2005. DOI: 10.1109/TAP.2005.844443,
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Differentiation of impedance[4]

$$W_{\mathrm{sto}} \propto \frac{\partial Z}{\partial \omega}$$

[4] M. Uzsoky and L. Solymár, "Theory of super-directive linear arrays", *Acta Physica Academiae Scientiarum Hungaricae*, vol. 6, no. 2, pp. 185–205, 1956. DOI: 10.1007/BF03157322,
R. F. Harrington, "Antenna excitation for maximum gain", *IEEE Trans. Antennas Propag.*, vol. 13, no. 6, pp. 896–903, 1965. DOI: 10.1109/TAP.1965.1138539,
D. Kajfez and W. P. Wheless, "Invariant definitions of the unloaded Q factor", *IEEE Antennas Propag. Magazine*, vol. 34, no. 7, pp. 840–841, 1986,
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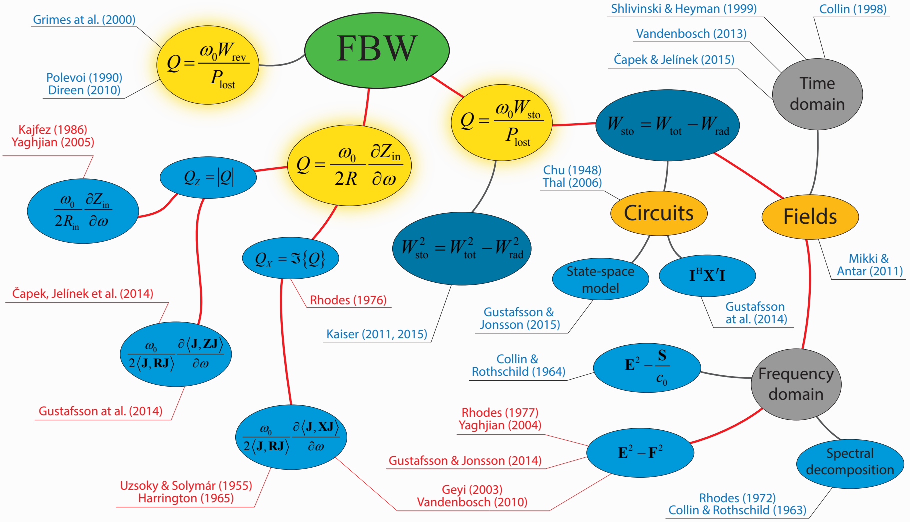
- coordinate dependent
- can be negative for $ka \gg 1$

Differentiation of impedance

$$W_{\text{sto}} \propto \frac{\partial Z}{\partial \omega}$$

- is not directly related to stored energy
- can be negative



Recent concepts of quality factor Q that will be discussed...

Source concept definition of Q

Pivotal concept derived by Vandenbosch[5]

$$Q^{(\mathrm{Van})} = \frac{\omega \left(W_{\mathrm{m}} + W_{\mathrm{e}} + W_{\mathrm{rad}}\right)}{P_{\mathrm{rad}}} \tag{1}$$

- can be negative, is coordinate independent, is gauge variant
- yields good results in most cases!

$$W_{\mathrm{m}} = \frac{1}{4\pi\epsilon_0\omega^2} \int\limits_{V_1}\int\limits_{V_2} k^2 \boldsymbol{J}\left(\boldsymbol{r}_1\right) \cdot \boldsymbol{J}^*\left(\boldsymbol{r}_2\right) \frac{\cos\left(kR\right)}{R} \,\mathrm{d}V_2 \,\mathrm{d}V_1$$

$$W_{\mathrm{e}} = \frac{1}{4\pi\epsilon_0\omega^2} \int\limits_{V_1}\int\limits_{V_2} \nabla_1 \cdot \boldsymbol{J}\left(\boldsymbol{r}_1\right) \nabla_2 \cdot \boldsymbol{J}^*\left(\boldsymbol{r}_2\right) \frac{\cos\left(kR\right)}{R} \,\mathrm{d}V_2 \,\mathrm{d}V_1$$

$$W_{\mathrm{rad}} = \frac{-k}{4\pi\epsilon_0\omega^2} \int\limits_{V_1}\int\limits_{V_2} \left(k^2 \boldsymbol{J}\left(\boldsymbol{r}_1\right) \cdot \boldsymbol{J}^*\left(\boldsymbol{r}_2\right) - \nabla_1 \cdot \boldsymbol{J}\left(\boldsymbol{r}_1\right) \nabla_2 \cdot \boldsymbol{J}^*\left(\boldsymbol{r}_2\right)\right) \sin\left(kR\right) \,\mathrm{d}V_2 \,\mathrm{d}V_1$$

[5] G. A. E. Vandenbosch, “Reactive energies, impedance, and Q factor of radiating structures”, *IEEE Trans. Antennas Propag.*, vol. 58, no. 4, pp. 1112–1127, 2010. DOI: 10.1109/TAP.2010.2041166.

Measurable Q factors: Q_X and Q_Z

Input reactance[6]

$$Q_X = \frac{\omega}{2R_{\mathrm{in}}} \left.\frac{\partial X_{\mathrm{in}}}{\partial \omega}\right|_{\partial I_0/\partial\omega=0} \tag{2}$$

or input impedance[7]

$$Q_Z = \frac{\omega}{2R_{\mathrm{in}}} \left.\frac{\partial Z_{\mathrm{in}}}{\partial \omega}\right|_{\partial I_0/\partial\omega=0} \tag{3}$$

- both concept represent *measurable Q* since they can be measured at input port

[6] R. F. Harrington, "Antenna excitation for maximum gain", *IEEE Trans. Antennas Propag.*, vol. 13, no. 6, pp. 896–903, 1965. DOI: 10.1109/TAP.1965.1138539, D. R. Rhodes, "Observable stored energies of electromagnetic systems", *J. Franklin Inst.*, vol. 302, no. 3, pp. 225–237, 1976. DOI: 10.1016/0016-0032(79)90126-1.

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Source concept definition of Q_X

Two recent attempts[8,9] to express Q_X in source concept formalism

$$Q_X^{(\mathrm{Cap})} = Q^{(\mathrm{Van})} + \frac{\omega W_{\partial\omega}}{P_{\mathrm{rad}}} \tag{4}$$

$$Q_X^{(\mathrm{Geyi})} = Q^{(\mathrm{Van})} + \frac{\omega W_{\mathrm{Geyi}}}{P_{\mathrm{rad}}} \tag{5}$$

$$W_{\partial\omega} = \frac{1}{4\pi\epsilon_0\omega} \int\limits_{V_1}\int\limits_{V_2} \Re\left\{ k^2 \frac{\partial \boldsymbol{J}(\boldsymbol{r}_1)\cdot\boldsymbol{J}^*(\boldsymbol{r}_2)}{\partial\omega} - \frac{\partial \nabla_1\cdot\boldsymbol{J}(\boldsymbol{r}_1)\,\nabla_2\cdot\boldsymbol{J}^*(\boldsymbol{r}_2)}{\partial\omega} \right\} \frac{\cos(kR)}{R}\,\mathrm{d}V_2\,\mathrm{d}V_1$$

$$W_{\mathrm{Geyi}} = \frac{1}{4\pi\epsilon_0\omega} \int\limits_{V_1}\int\limits_{V_2} \Im\left\{ k^2 \frac{\partial \boldsymbol{J}(\boldsymbol{r}_1)\cdot\boldsymbol{J}^*(\boldsymbol{r}_2)}{\partial\omega} - \frac{\partial \nabla_1\cdot\boldsymbol{J}(\boldsymbol{r}_1)\,\nabla_2\cdot\boldsymbol{J}^*(\boldsymbol{r}_2)}{\partial\omega} \right\} \frac{\sin(kR)}{R}\,\mathrm{d}V_2\,\mathrm{d}V_1$$

[8] M. Capek, L. Jelinek, P. Hazdra, *et al.*, "The measurable Q factor and observable energies of radiating structures", *IEEE Trans. Antennas Propag.*, vol. 62, no. 1, pp. 311–318, 2014. DOI: 10.1109/TAP.2013.2287519, W. Geyi, "On stored energies and radiation Q", , *IEEE Trans. Antennas Propag.*, vol. 63, no. 2, pp. 636–645, 2015. DOI: 10.1109/TAP.2014.2384028.

[9] Note that **there is nowhere** stated that $Q_X^{(\mathrm{Cap})}$ has any relationship to the stored energy.

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Another concepts of Q_X

Another concepts[10] are recalled to decide whether $Q_X^{(\mathrm{Cap})}$ or $Q_X^{(\mathrm{Geyi})}$ is correct:

$$Q^{(\mathrm{Yagh})} = Q_X - \frac{\omega W_{\partial F}}{P_{\mathrm{rad}}} \tag{6}$$

$$Q^{(\mathrm{Gust})} = Q^{(\mathrm{Van})} + \frac{\omega W_{F_2}}{P_{\mathrm{rad}}} \tag{7}$$

$$W_{\partial F} = \frac{2}{Z_0} \oint\limits_{S_\infty} \Im \left\{ \boldsymbol{F}^* \left(\boldsymbol{r}\right) \cdot \frac{\partial \boldsymbol{F}\left(\boldsymbol{r}\right)}{\partial \omega} \right\} \sin\left(\vartheta\right) \,\mathrm{d}\vartheta \,\mathrm{d}\varphi$$

$$W_{F_2} = \frac{1}{4\pi\epsilon_0\omega^2} \int\limits_{V_1} \int\limits_{V_2} \Im \left\{ k^2 \boldsymbol{J}\left(\boldsymbol{r}_1\right) \cdot \boldsymbol{J}^*\left(\boldsymbol{r}_2\right) - \nabla_1 \cdot \boldsymbol{J}\left(\boldsymbol{r}_1\right) \nabla_2 \cdot \boldsymbol{J}^*\left(\boldsymbol{r}_2\right) \right\} \mathcal{G}\left(\boldsymbol{r}_1, \boldsymbol{r}_2\right) \mathrm{d}V_2 \,\mathrm{d}V_1$$

$$\mathcal{G}\left(\boldsymbol{r}_1, \boldsymbol{r}_2\right) = \frac{k^2 \left(\|\boldsymbol{r}_1\|^2 - \|\boldsymbol{r}_2\|^2\right) \mathrm{j}_1\left(kR\right)}{R}$$

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[10] M. Gustafsson and B. L. G. Jonsson, "Stored electromagnetic energy and antenna Q", , *Prog. Electromagn. Res.*, vol. 150, pp. 13–27, 2014, A. D. Yaghjian and S. R. Best, "Impedance, bandwidth and Q of antennas", *IEEE Trans. Antennas Propag.*, vol. 53, no. 4, pp. 1298–1324, 2005. DOI: 10.1109/TAP.2005.844443.

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Independent test of Q_X

Combining previous relations, we arrive at

$$\boxed{Q_X = Q^{(\mathrm{Van})} + \frac{\omega W_{\partial F}}{P_{\mathrm{rad}}} + \frac{\omega W_{F_2}}{P_{\mathrm{rad}}}. \qquad (8)}$$

Observations:

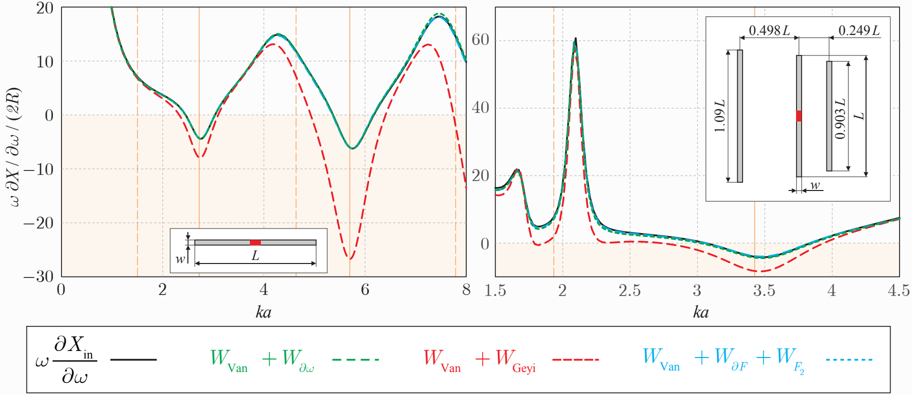
- (8) forms second independent test of $Q_X^{(\mathrm{Cap})}$ (4) and $Q_X^{(\mathrm{Geyi})}$ (5) being equal to Q_X
- Q_X and $Q^{(\mathrm{Van})}$ are not coordinate dependent
- $W_{\partial F}$ and W_{F_2} are coordinate dependent

Based on (8), the following expression[11] should be inspected (RHS is from e.q. (65), $\boldsymbol{J}' \equiv \partial \boldsymbol{J}(\boldsymbol{r}) / \partial \omega$):

$$W_{\partial F} + W_{F_2} = \frac{1}{4} \oint\limits_{S_\infty} \Im \left\{ \boldsymbol{E}\left(\boldsymbol{J}'\right) \times \boldsymbol{H}^*\left(\boldsymbol{J}\right) - \boldsymbol{E}\left(\boldsymbol{J}\right) \times \boldsymbol{H}^*\left(\boldsymbol{J}'\right) \right\} \cdot \mathbf{n}_0 \, \mathrm{d}S \tag{9}$$

[11] G. A. E. Vandenbosch, “Reactive energies, impedance, and Q factor of radiating structures”, *IEEE Trans. Antennas Propag.*, vol. 58, no. 4, pp. 1112–1127, 2010. DOI: 10.1109/TAP.2010.2041166.

Comparison of all methods



Reply to these results has been posted on arXiv[12].

[12] W. Geyi, “Reply to Comments on ‘Stored energies and radiation Q’”, , 2015. eprint: 1509.07252. [Online]. Available: http://arxiv.org/abs/1509.07252.

Results

Correct[13] source definition of Q_X is

$$\boxed{Q_X = Q_X^{(\mathrm{Cap})}}$$

thus, based on previous comparison, we have:

$$W_{\partial\omega} = W_{\partial F} + W_{F_2}. \tag{10}$$

- $W_{\partial\omega}$ is not coordinate dependent, but $W_{\partial F}$ and W_{F_2} are.

Based on (10), the following expression[14] should be inspected (RHS is from e.q. (65), $\boldsymbol{J}' \equiv \partial \boldsymbol{J}(\boldsymbol{r}) / \partial\omega$):

$$W_{\partial\omega} = \frac{1}{4} \oint\limits_{S_\infty} \Im \left\{ \boldsymbol{E}(\boldsymbol{J}') \times \boldsymbol{H}^*(\boldsymbol{J}) - \boldsymbol{E}(\boldsymbol{J}) \times \boldsymbol{H}^*(\boldsymbol{J}') \right\} \cdot \mathbf{n}_0 \, \mathrm{d}S \tag{11}$$

[13] M. Capek and L. Jelinek, "Comments on 'On Stored Energies and Radiation Q'", , *IEEE Trans. Antennas Propag.*, 2015.

[14] G. A. E. Vandenbosch, "Reactive energies, impedance, and Q factor of radiating structures", *IEEE Trans. Antennas Propag.*, vol. 58, no. 4, pp. 1112–1127, 2010. DOI: 10.1109/TAP.2010.2041166.

Important Remarks

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…there is no coherent definition of the stored energy based no selected approaches.

As a consequence, new paradigm should probably be looked for…

What the stored energy is?

Proposed definition of stored energy[15]

Stored electromagnetic energy is that part of the total electromagnetic energy that is, in comparison with the radiated energy, bound to the sources of the field, being unable to escape towards infinity.

- In all cases, the statement above can symbolically be written as

$$W_{\mathrm{sto}} = \mathcal{F}\left(W_{\mathrm{tot}}, W_{\mathrm{rad}}\right). \tag{12}$$

- In order to correctly define stored energy W_{sto}, radiation energy W_{rad} has to be completely understood.
  - Expression (12) explicitly defines the radiated energy[15].

[15] M. Capek and L. Jelinek, "Various interpretations of the stored and the radiated energy density", , 2015. [Online]. Available: http://arxiv.org/abs/1503.06752

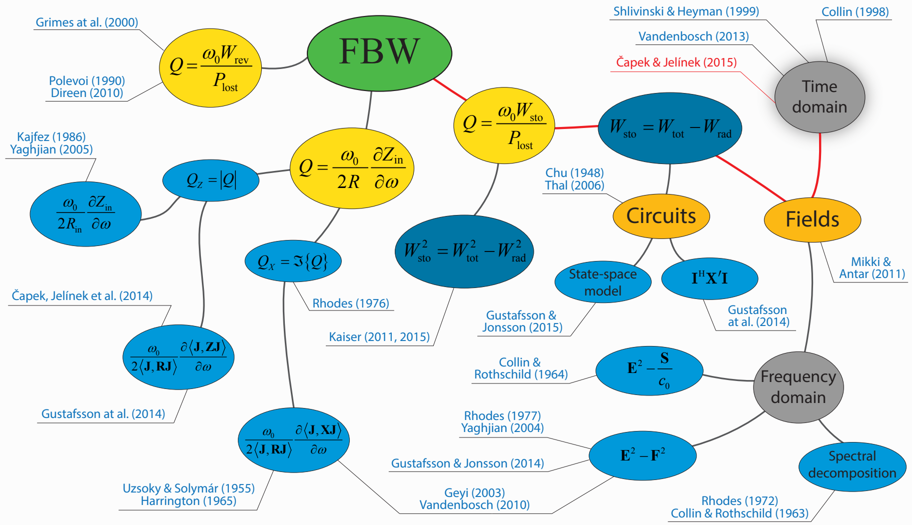
What conditions should be met?

$$\text{Stored energy} \stackrel{?}{=} \text{Physical quantity}$$

As a physical quantity[16], stored energy has to embody i.a.

- uniqueness,
- positive semi-definiteness,
- gauge invariance,
- coordinate-independence,
- equality to total energy for PEC cavities.

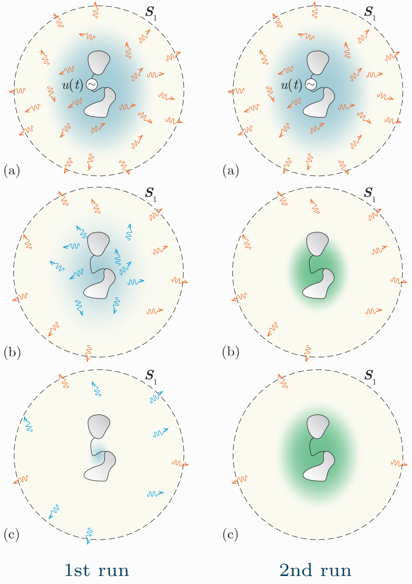
[16] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. John Wiley, 1998



- Novel scheme[17] satisfies all requirements for stored energy.

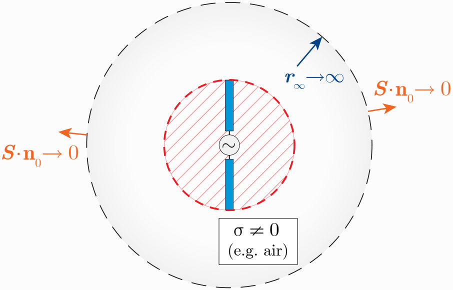
[17] M. Capek, L. Jelinek, and G. A. E. Vandenbosch, "Stored electromagnetic energy and quality factor of radiating structures", , 2015. [Online]. Available: http://arxiv.org/abs/1403.0572

Novel idea how to evaluate stored energy



- Based on several thought experiments:
  - the only measure of stored energy is the radiated energy,
  - two runs of the method are required (difference yields the stored energy inside circumscribing sphere),
- presented at EuCAP 2015 and detailed at APS 2015,
- details can be found on arXive[17]
  - manuscript submitted to Proc. Royal Soc. A.

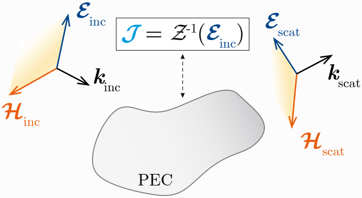
(Advanced) Contentious issues #1



We have practically no far-field for $\epsilon_r > 1$.

- Energy stored by a scatterer[18]
  - There is no feeding port.

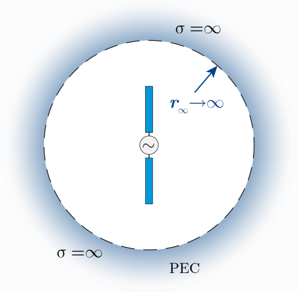
- Definition of radiated energy in dissipative environment[18]
  - Radiation cannot be defined as a part of total energy that reached the far-field ($\sigma \neq 0 \Rightarrow \boldsymbol{E}_{\text{far}} \times \boldsymbol{H}^*_{\text{far}} = 0$).



Scatterer without any input port.

[18] Questions raised during several discussions with Mats Gustafsson, Lars Jonsson and Lukas Jelinek.

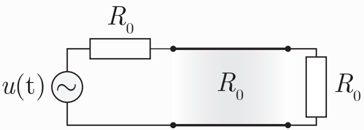
(Advanced) Contentious issues #2



Infinity large PEC cavity.

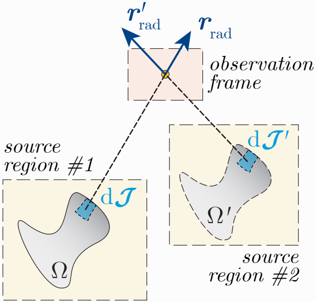
- Is there any radiation for a PEC cavity of radius $r \to \infty$?
  - This is combination of several tough problems mentioned above.

- Physical meaning of radiation
  - Being e.g. inside a perfectly matched transmission line the outgoing power should be considered as radiated power.



Perfectly matched line ≡ "free space"?

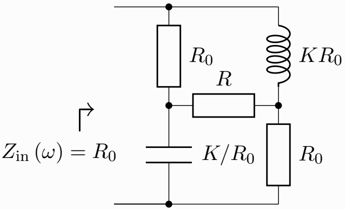
(Advanced) Contentious issues #3



Radiation cannot be determined locally.

- Circuit's synthesis
  - Is (Brune) synthesis unique?
  - What about hidden states?

- Locality[18]
  - *There is no local definition of outgoing directions.*[19] Thus, the amount of radiated energy may not be interpreted strictly locally.



Zobel's network

[18] Questions raised during several discussions with Mats Gustafsson, Lars Jonsson and Lukas Jelinek.

[19] From correspondence with Gerald Kaiser.

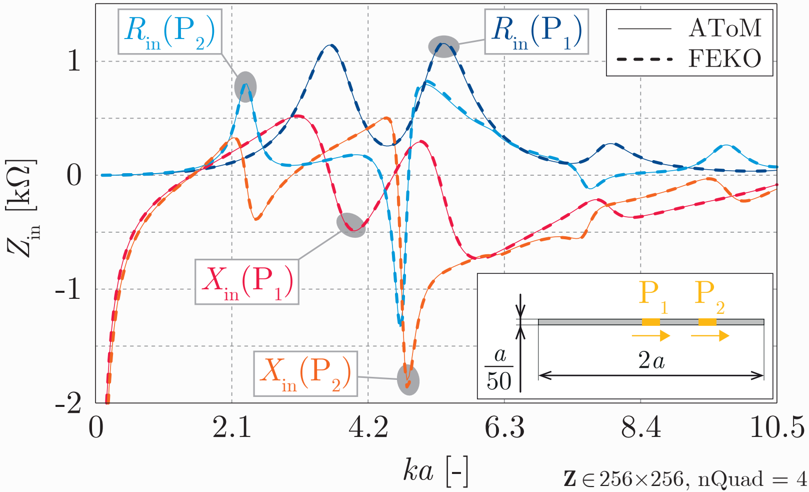
Implementation of stored energy evaluation

ANTENNA TOOLBOX FOR MATLAB

- antennatoolbox.com
- all in Matlab
  - source concept[20] (charact. modes, optimization, post-processing)
  - various techniques to evaluate stored energy will be available
- YouTube channel
  - AToM's core is almost complete
  - numerical methods (MoM, BEM, CM) are now implemented

[20] See also the presentation from COST VISTA meeting in Madrid.

AToM 2D EFIE+MFIE MoM (Matlab)



$\mathbf{Z} \in 256\times256$, nQuad $= 4$

Preliminary results for simple structure (comparison with FEKO).

Thank you!

For complete PDF presentation see ‣ capek.elmag.org

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