Optimal Currents and Shape Synthesis in Electromagnetism

Part II – Topology Sensitivity

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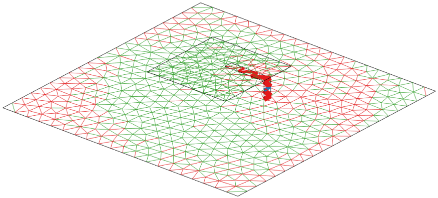
Chalmers University of Technology

Outline

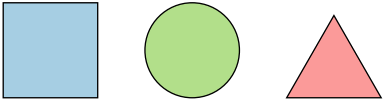


Topology sensitivity of a PIFA.

This talk concerns:

- electric currents in vacuum,
- time-harmonic quantities, *i.e.*, $\boldsymbol{\mathcal{A}}(\boldsymbol{r}, t) = \mathrm{Re}\left\{\boldsymbol{A}(\boldsymbol{r})\exp(\mathrm{j}\omega t)\right\}$.

Analysis × Synthesis



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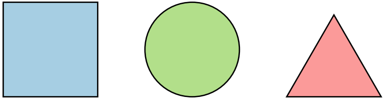

Analysis ($\mathcal{A}$)

- Shape Ω is given, BCs are known, determine EM quantities.

$$g = \mathcal{L}\left\{\boldsymbol{J}\left(\boldsymbol{r}\right)\right\} = \mathcal{A}f$$

$$f \equiv \left\{\Omega, \boldsymbol{E}^{\mathrm{i}}\right\},\ g \equiv \left\{p\right\}$$

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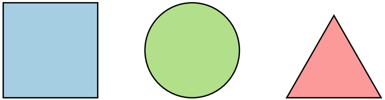

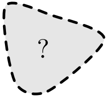

Analysis ($\mathcal{A}$)

- Shape Ω is given, BCs are known, determine EM quantities.

$$g = \mathcal{L}\left\{\boldsymbol{J}\left(\boldsymbol{r}\right)\right\} = \mathcal{A}f$$

Synthesis ($\mathcal{S} \equiv \mathcal{A}^{-1}$)

- EM behavior is specified, neither Ω nor BCs are known.

$$f = \mathcal{S}g = \mathcal{A}^{-1}g$$

$$f \equiv \left\{\Omega, \boldsymbol{E}^{\mathrm{i}}\right\}, \; g \equiv \{p\}$$

Synthesis

How to get $f = \mathcal{A}^{-1} g$?

Questions inherently related to synthesis are[1] ($f \equiv \{\Omega, \boldsymbol{E}^{\mathrm{i}}\}$, $g \equiv \{p_i\}$)

1. Can g be chosen arbitrary?
2. If g is such that there exists a solution f, is that solution unique?
3. If g is known only approximately, which is always the case, is the corresponding solution for f close to the exact one?
4. If f is not exactly realized what effect will this have on $\mathcal{A} f$?

[1] G. Deschamps and H. Cabayan, "Antenna synthesis and solution of inverse problems by regularization methods," *IEEE Transactions on Antennas and Propagation*, vol. 20, no. 3, pp. 268–274, 1972. DOI: 10.1109/tap.1972.1140197. [Online]. Available: https://doi.org/10.1109/tap.1972.1140197

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3. If g is known only approximately, which is always the case, is the corresponding solution for f close to the exact one? No.
4. If f is not exactly realized what effect will this have on $\mathcal{A} f$? Potentially huge.

Generally, infinitely many possibilities and local minima → need for shape discretization.

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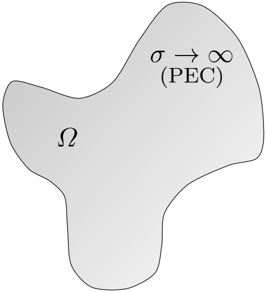
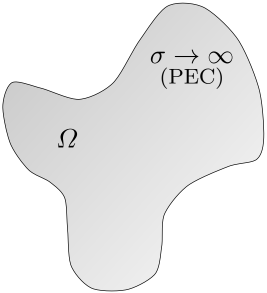
Discretization



Original problem.

Discretization



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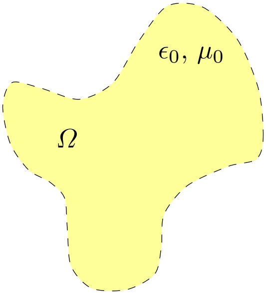
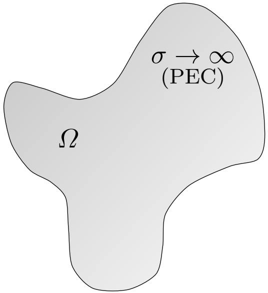


Equivalent problem.

Discretization



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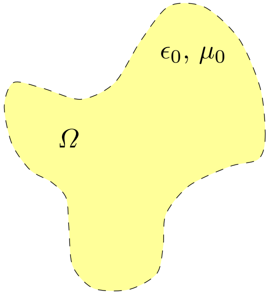

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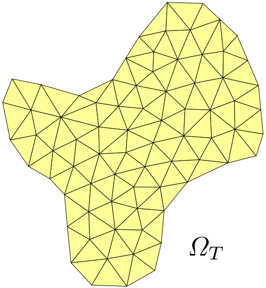

Triangularized domain Ω_T.

Structure $\Omega \to \Omega_T$, current density in vacuum $\boldsymbol{J}(\boldsymbol{r})$, $\boldsymbol{r} \in \Omega_T$.

Operators Represented In RWG Basis Functions

Starting point in this work is a given discretization into T triangles t_i, $\Omega_T = \bigcup_{i=1}^{T} t_i$.

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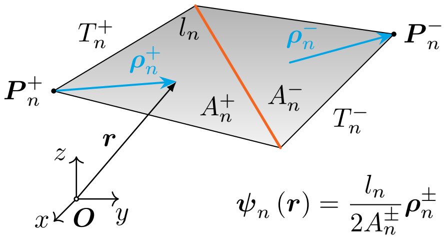
RWG basis functions $\{\boldsymbol{\psi}_n(\boldsymbol{r})\}$ are applied as

$$\boldsymbol{J}(\boldsymbol{r}) \approx \sum_{n=1}^{N} I_n \boldsymbol{\psi}_n(\boldsymbol{r}),$$

where N is the number of all inner edges.



RWG basis function $\boldsymbol{\psi}_n(\boldsymbol{r})$.

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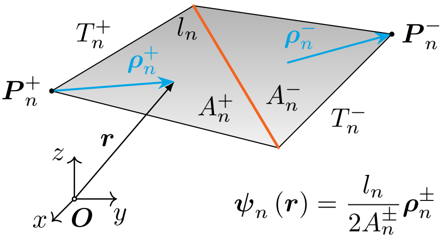
Matrix representation of the operators used

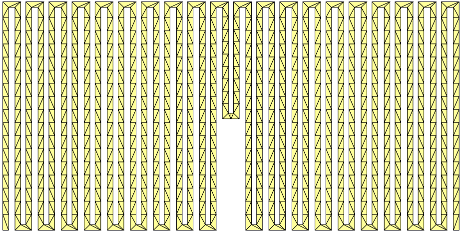
$$\langle \boldsymbol{J}, \mathcal{A}\boldsymbol{J} \rangle = [I_m^* \langle \boldsymbol{\psi}_m, \mathcal{A}\boldsymbol{\psi}_n \rangle I_n] = \mathbf{I}^{\mathrm{H}} \mathbf{A} \mathbf{I}.$$

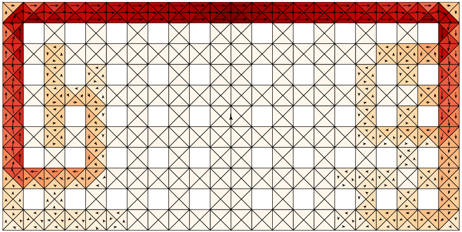


RWG basis function $\boldsymbol{\psi}_n(\boldsymbol{r})$.

Shape Synthesis: Properties and Approaches

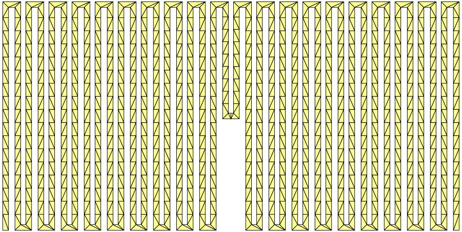
1. Designers' skill and knowledge.
2. Parametric sweeps.
3. Heuristic algorithms (global optimization).
4. Topology optimization (local optimization).

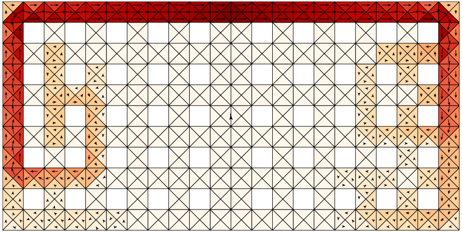




Shape Synthesis: Properties and Approaches

1. Designers' skill and knowledge.
   - Nonintuitive/complex design?
2. Parametric sweeps.
   - What parameters? How many?
3. Heuristic algorithms (global optimization).
   - Convergence. No-free-lunch. "Solution."
4. Topology optimization (local optimization).
   - This talk... partly.





Optimal solution:

- Combination of all approaches.

Topology Optimization

$$\text{minimize} \quad f = \int\limits_{\Omega} F\left(\rho\left(\boldsymbol{r}\right)\right) \mathrm{d}V$$

$$\text{subject to} \quad \int\limits_{\Omega} \rho \, \mathrm{d}V - V_0 \leq 0$$

- min. compliance → max. stiffness
- solved within FEM
- mesh dependence
- instability (chess board)

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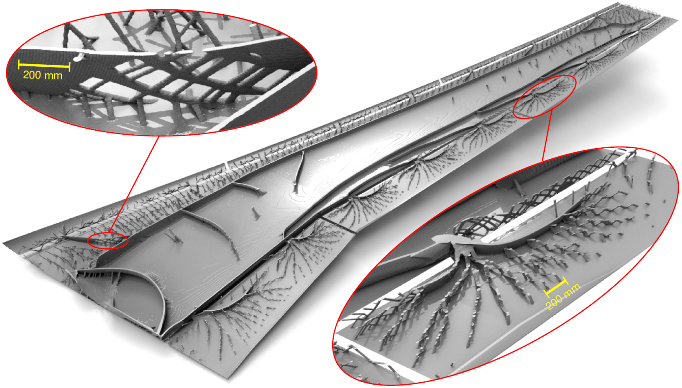


$1216 \times 3456 \times 256 \approx 1.1 \cdot 10^9$ unknowns, FEM[2].

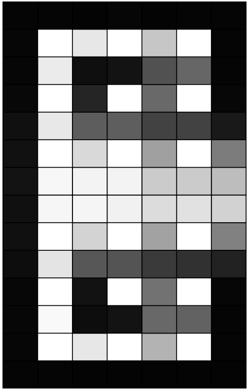
[2] N. Aage, E. Andreassen, B. S. Lazarov, *et al.*, "Giga-voxel computational morphogenesis for structural design," *Nature*, vol. 550, pp. 84–86, 2017. DOI: 10.1038/nature23911

Topology Optimization in EM

State-of-the-art in mechanics, serious problems in EM[3]

- "gray" elements, rounding yields different results,
- numerical oscillation (chessboard),
- more sensitive to local minima (current paths?),
- threshold function for MoM.

Fundamental difference between EM vector field and stiffness in mechanics?



Histogram of the best candidates found for $\min_{\mathbf{I}} Q$, NSGA-II.

[3]S. Liu, Q. Wang, and R. Gao, "A topology optimization method for design of small GPR antennas," *Struct. Multidisc. Optim.*, vol. 50, pp. 1165–1174, 2014. DOI: 10.1007/s00158-014-1106-y

Topology Sensitivity

Idea behind this work

Let us accept NP-hardness of the problem, do brute force, but do it cleverly...

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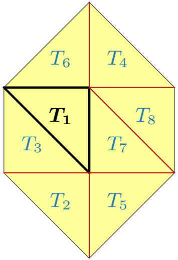
Idea behind this work

Let us accept NP-hardness of the problem, do brute force, but do it cleverly...

- Inspired by pixeling[4], but RWG functions are the unknowns (T vs. N unknowns).
- Fixed mesh grid Ω_T: operators calculated once, results comparable with the bounds.
- Woodbury identity employed: get rid of repetitive matrix inversion!
- Feeding is specified at the beginning.

[4] Y. Rahmat-Samii, J. M. Kovitz, and H. Rajagopalan, "Nature-inspired optimization techniques in communication antenna design," *Proc. IEEE*, vol. 100, no. 7, pp. 2132–2144, 2012. DOI: 10.1109/JPROC.2012.2188489

Comparison of Pixeling Techniques



Pixel removal

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & Z_{23} & \cdots & Z_{2N} \\ Z_{31} & Z_{32} & Z_{33} & \cdots & Z_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & Z_{N3} & \cdots & Z_{NN} \end{bmatrix}$$

Classical pixeling removes metallic patches[5].

$$\left(\mathbf{Z_G} + \mathbf{Z}_\mathrm{L}\right)\mathbf{I} = \mathbf{Z}\mathbf{I} = \mathbf{V}$$

[5] Y. Rahmat-Samii, J. M. Kovitz, and H. Rajagopalan, "Nature-inspired optimization techniques in communication antenna design," *Proc. IEEE*, vol. 100, no. 7, pp. 2132–2144, 2012. DOI: 10.1109/JPROC.2012.2188489

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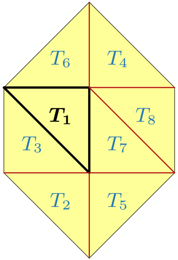

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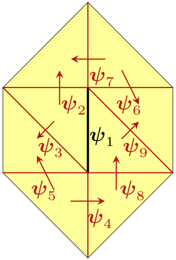

Edge removal

$$\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & Y_{22} & Y_{23} & \cdots & Y_{2N} \\ 0 & Y_{32} & Y_{33} & \cdots & Y_{2N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & Y_{N2} & Y_{N3} & \cdots & Y_{NN} \end{bmatrix}$$

Proposed basis function removal.

$$\mathbf{I} = \mathbf{Z}^{-1}\mathbf{V} = \mathbf{YV}$$

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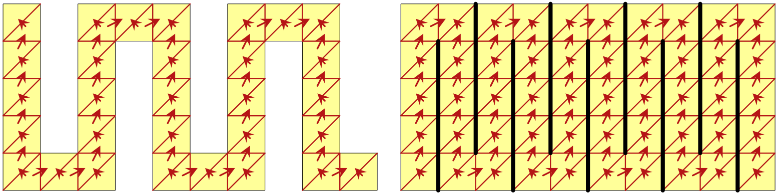
Pixeling and Edge Removal



Comparison of the longest meander possible for classical pixeling and edge removal.

- "Infinitesimally" small perturbation of a structure Ω_T is a removal of RWG edge[6].

[6] M. Capek, L. Jelinek, and M. Gustafsson, "Shape synthesis based on topology sensitivity," , 2018, submitted, arxiv: 1808.02479. [Online]. Available: https://arxiv.org/abs/1808.02479

Shape Synthesis: Rigorous Definition

For a given impedance matrix $\mathbf{Z} \in \mathbb{C}^{N \times N}$, matrices $\mathbf{A}$, $\{\mathbf{B}_i\}$, $\{\mathbf{B}_j\}$, given excitation vector $\mathbf{V} \in \mathbb{C}^{N}$, found a vector $\boldsymbol{x}$ such that

$$\begin{aligned} \text{minimize} \quad & \mathbf{I}^{\mathrm{H}} \mathbf{A} (\boldsymbol{x}) \mathbf{I} \\ \text{subject to} \quad & \mathbf{I}^{\mathrm{H}} \mathbf{B}_i (\boldsymbol{x}) \mathbf{I} = p_i \\ & \mathbf{I}^{\mathrm{H}} \mathbf{B}_j (\boldsymbol{x}) \mathbf{I} \leq p_j \\ & \mathbf{Z} (\boldsymbol{x}) \mathbf{I} = \mathbf{V} \\ & \boldsymbol{x} \in \{0, 1\}^N \end{aligned}$$

- structure perturbation
- combinatorial optimization
- $\widehat{\mathbf{A}} = \left(\boldsymbol{x} * \boldsymbol{x}^{\mathrm{T}}\right) \otimes \mathbf{A}$

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- material perturbation
- relaxation of the combinatorial approach
- $\widehat{A}_{ii} = A_{ii} + x_i R_0$

Incorporation of Lumped Element R_∞

$$\left(\mathbf{Z}_{\mathbf{G}} + \mathbf{Z}_{\mathrm{L}}\right)\mathbf{I} = \mathbf{Z}\mathbf{I} = \mathbf{V}$$

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Lumped element with resistivity R_∞

$$Z_{\mathrm{L},nn} = R_\infty \quad \Leftrightarrow \quad n \in \mathcal{B}$$

Example:

$$\mathcal{B} = \{1, 3\}$$

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$$C_{\mathcal{B},nn} = \begin{cases} 0 & \Leftrightarrow \quad n \notin \mathcal{B} \\ 1 & \Leftrightarrow \quad \text{otherwise} \end{cases}$$

(All columns containing only zeros are removed.)

Example:

$$\mathcal{B} = \{1, 3\}$$

$$\mathbf{C}_\mathcal{B} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \end{bmatrix}^\mathrm{T}$$

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$$\mathbf{Z}_\mathrm{L} = \mathbf{C}_\mathcal{B} R_\infty \mathbf{C}_\mathcal{B}^\mathrm{T},$$

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$$\mathbf{C}_\mathcal{B} R_\infty \mathbf{C}_\mathcal{B}^\mathrm{T} = \begin{bmatrix} R_\infty & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & R_\infty & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

Utilization of Woodbury Formula

Definitions

$$\mathbf{Z} = \mathbf{Z}_{\mathbf{G}} + \mathbf{Z}_{\mathrm{L}} = \mathbf{Z}_{\mathbf{G}} + \mathbf{C}_{\mathcal{B}} R_{\infty} \mathbf{C}_{\mathcal{B}}^{\mathrm{T}}, \qquad \mathbf{I} = \mathbf{Z}^{-1}\mathbf{V} = \mathbf{Y}\mathbf{V}.$$

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Sherman-Morrison-Woodbury formula

$$\left(\mathbf{A} + \mathbf{E}\mathbf{B}\mathbf{F}\right)^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{E}\left(\mathbf{B}^{-1} + \mathbf{F}\mathbf{A}^{-1}\mathbf{E}\right)^{-1}\mathbf{F}\mathbf{A}^{-1}$$

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$$\mathbf{Y} = \mathbf{Z}^{-1} = \mathbf{Z}_{\mathbf{G}}^{-1} - \mathbf{Z}_{\mathbf{G}}^{-1}\mathbf{C}_{\mathcal{B}}\left(\frac{1}{R_{\infty}}\mathbf{1}_{D} + \mathbf{C}_{\mathcal{B}}^{\mathrm{T}}\mathbf{Z}_{\mathbf{G}}^{-1}\mathbf{C}_{\mathcal{B}}\right)^{-1}\mathbf{C}_{\mathcal{B}}^{\mathrm{T}}\mathbf{Z}_{\mathbf{G}}^{-1}$$

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For $\mathbf{Z}_{\mathbf{G}}^{-1} = \mathbf{Y}_{\mathbf{G}}$ and $R_{\infty} \to \infty$

$$\mathbf{Y} = \mathbf{Y}_{\mathbf{G}} - \mathbf{Y}_{\mathbf{G}}\mathbf{C}_{\mathcal{B}}\left(\mathbf{C}_{\mathcal{B}}^{\mathrm{T}}\mathbf{Y}_{\mathbf{G}}\mathbf{C}_{\mathcal{B}}\right)^{-1}\mathbf{C}_{\mathcal{B}}^{\mathrm{T}}\mathbf{Y}_{\mathbf{G}}.$$

Simplification With Indexing Property of Matrix $\mathbf{C}_{\mathcal{B}}$

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For one (n-th) edge removed, $|\mathcal{B}| = 1$:

$$\mathbf{Y}_\mathbf{G} \mathbf{C}_\mathcal{B} = \mathbf{y}_{\mathbf{G},n}, \qquad \left(\mathbf{C}_\mathcal{B}^\mathrm{T} \mathbf{Y}_\mathbf{G} \mathbf{C}_\mathcal{B} \right)^{-1} = \frac{1}{Y_{nn}}, \qquad \mathbf{C}_\mathcal{B}^\mathrm{T} \mathbf{Y}_\mathbf{G} = \mathbf{y}_{\mathbf{G},n}^\mathrm{T}.$$

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Notice $\mathbf{C}_\mathcal{B}$ is indexing matrix (MATLAB) only…

$$\mathbf{Y} = \mathbf{Y}_\mathbf{G} - \frac{\mathbf{y}_{\mathbf{G},n}\mathbf{y}_{\mathbf{G},n}^\mathrm{T}}{Y_{nn}}.$$

Incorporation of Fixed and Localized Feeding

Imagine further, that only one (f-th) edge is fed

$$\mathbf{V}_f = V_0 \begin{bmatrix} 0 & \cdots & l_f & \cdots & 0 \end{bmatrix}^{\mathrm{T}}.$$

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$$\mathbf{I}_{fn} = \left(\mathbf{Y}_{\mathbf{G}} - \frac{\mathbf{y}_{\mathbf{G},n} \mathbf{y}_{\mathbf{G},n}^{\mathrm{T}}}{Y_{nn}} \right) \mathbf{V}_f = \cdots = \mathbf{I}_f - \left(\frac{l_f l_n}{l_n^2} \frac{Y_{fn}}{Y_{nn}} \right) V_0 l_n \mathbf{y}_{\mathbf{G},n} = \mathbf{I}_f + \zeta_{fn} \mathbf{I}_n,$$

with $\mathbf{I}_f = \mathbf{Y}_{\mathbf{G}} \mathbf{V}_f$ and

$$\zeta_{ij} = - \frac{l_i l_j}{l_j^2} \frac{Y_{ij}}{Y_{jj}} = - \frac{Z_{\mathrm{in},jj}}{Z_{\mathrm{in},ij}}.$$

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This is equivalent to a specific two-port feeding

$$\mathbf{V} = V_0 \begin{bmatrix} 0 & \dots & l_f & \dots & \zeta_{fn} l_n & \dots & 0 \end{bmatrix}^{\mathrm{T}}.$$

Topology Sensitivity

All potential removals at once:

$$\mathbf{I}_{f\mathcal{B}} = \begin{bmatrix} \mathbf{I}_f + \zeta_{f1}\mathbf{I}_1 & \cdots & \mathbf{I}_f + \zeta_{fN}\mathbf{I}_N \end{bmatrix}.$$

Topology Sensitivity

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An antenna observable defined as quadratic form

$$x(\mathbf{I}) = \frac{\mathbf{I}^{\mathrm{H}}\mathbf{A}\mathbf{I}}{\mathbf{I}^{\mathrm{H}}\mathbf{B}\mathbf{I}}.$$

is calculated with a Hadamard product (vectorization)

$$\mathbf{x}(\mathbf{I}_{f\mathcal{B}}) = \operatorname{diag}\left(\mathbf{I}_{f\mathcal{B}}^{\mathrm{H}}\mathbf{A}\mathbf{I}_{f\mathcal{B}}\right) \oslash \operatorname{diag}\left(\mathbf{I}_{f\mathcal{B}}^{\mathrm{H}}\mathbf{B}\mathbf{I}_{f\mathcal{B}}\right).$$

Topology Sensitivity

All potential removals at once:

$$\mathbf{I}_{f\mathcal{B}} = \left[\; \mathbf{I}_f + \zeta_{f1}\mathbf{I}_1 \quad \cdots \quad \mathbf{I}_f + \zeta_{fN}\mathbf{I}_N \; \right].$$

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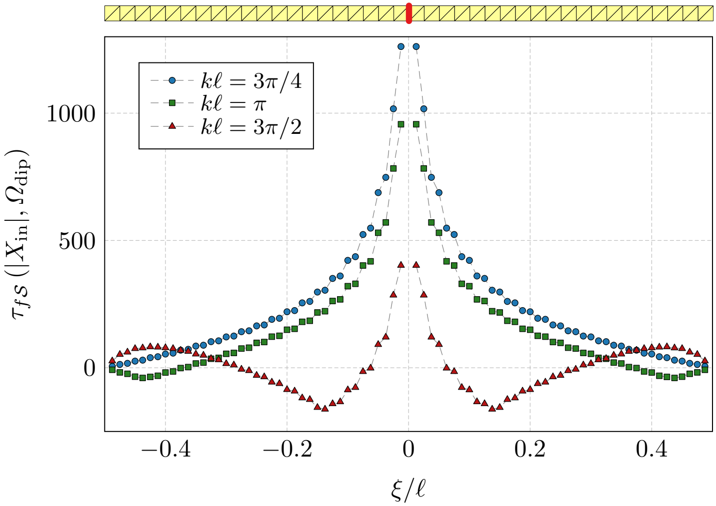
$$\mathbf{x}\left(\mathbf{I}_{f\mathcal{B}}\right) = \mathrm{diag}\left(\mathbf{I}_{f\mathcal{B}}^{\mathrm{H}}\mathbf{A}\mathbf{I}_{f\mathcal{B}}\right) \oslash \mathrm{diag}\left(\mathbf{I}_{f\mathcal{B}}^{\mathrm{H}}\mathbf{B}\mathbf{I}_{f\mathcal{B}}\right).$$

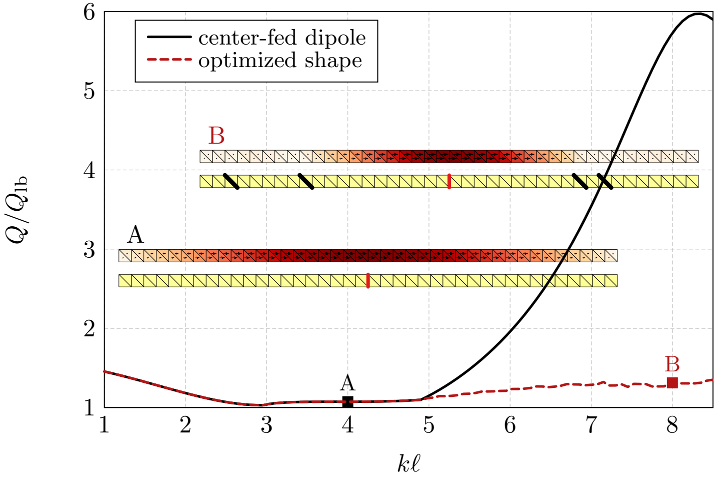
Finally, topology sensitivity is defined here as

$$\boldsymbol{\tau}_{f\mathcal{B}}\left(x, \varOmega_T\right) = \mathbf{x}\left(\mathbf{I}_{f\mathcal{B}}\right) - x\left(\mathbf{I}_f\right) \approx \nabla \boldsymbol{x}\left(\mathbf{I}_f\right).$$

Example: Thin-strip Dipole – Input Reactance



Example: Thin-strip Dipole – Q-factor

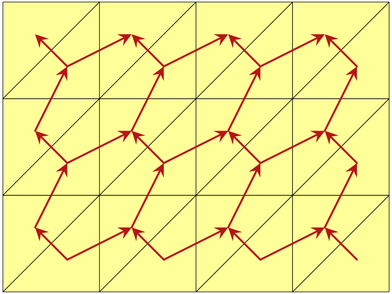


Radiation Q-factor of center-fed dipole Ω_{dip}, discretized into $N = 79$ basis functions.

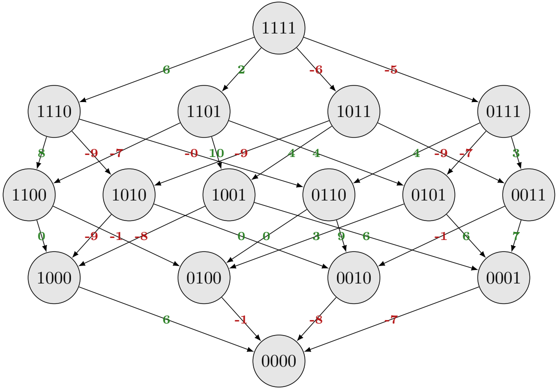
Greedy Step

A discretization establishes a graph.

$$G\left(V,E\right)=G\left(\mathbf{P},E\right)\to\left\{T_i\right\}\to\left\{\boldsymbol{\psi}_n\left(\boldsymbol{r}\right)\right\}$$

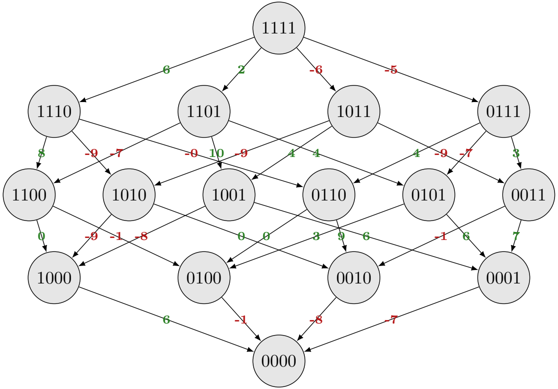


Graph Representation: Reduction to Tree



Synthesis for $N = 4$ as a directional binary tree.

Graph Representation: Reduction to Tree



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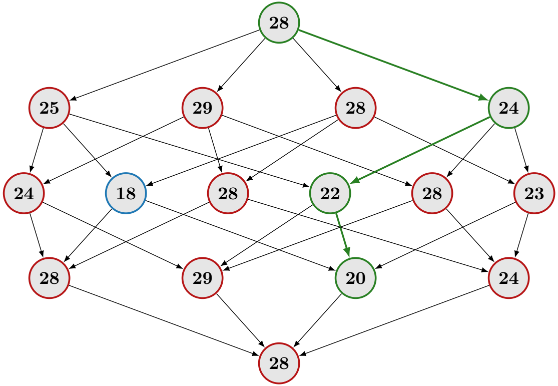


Greedy algorithm in directional graph.

Greedy Algorithm

One gradient-based search through the entire tree (the most pessimistic run):

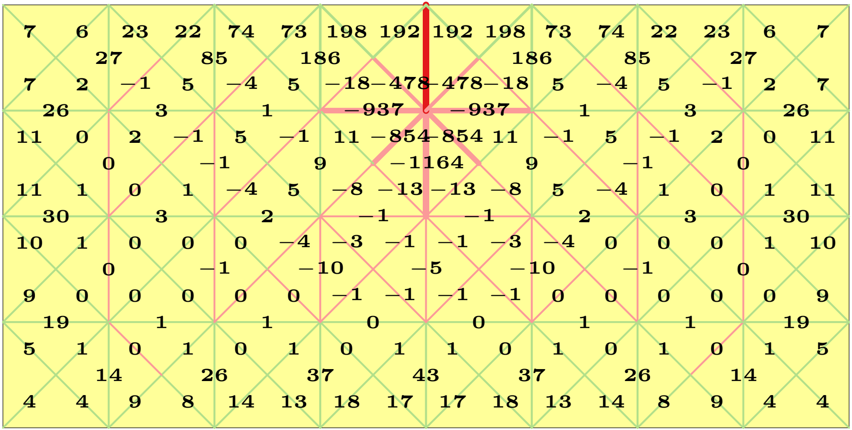
- max $(N-1)$ series
- $N\,(N-1)\,(N-2)\cdots = N!$ evaluations

Shermann-Morrison-Woodbury: $N-n$ speed-up at every tree level

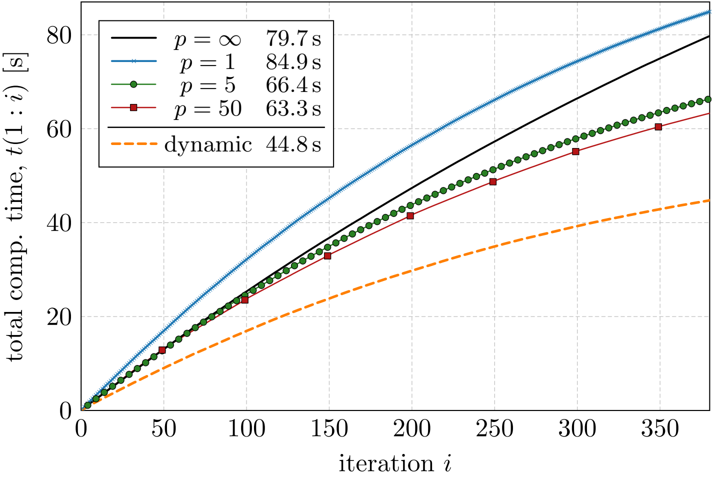
Note of solvability of the problem

Problem is not convex $\rightarrow$ combination of global and local algorithms.

Greedy Algorithm – Example: Rectangular Plate



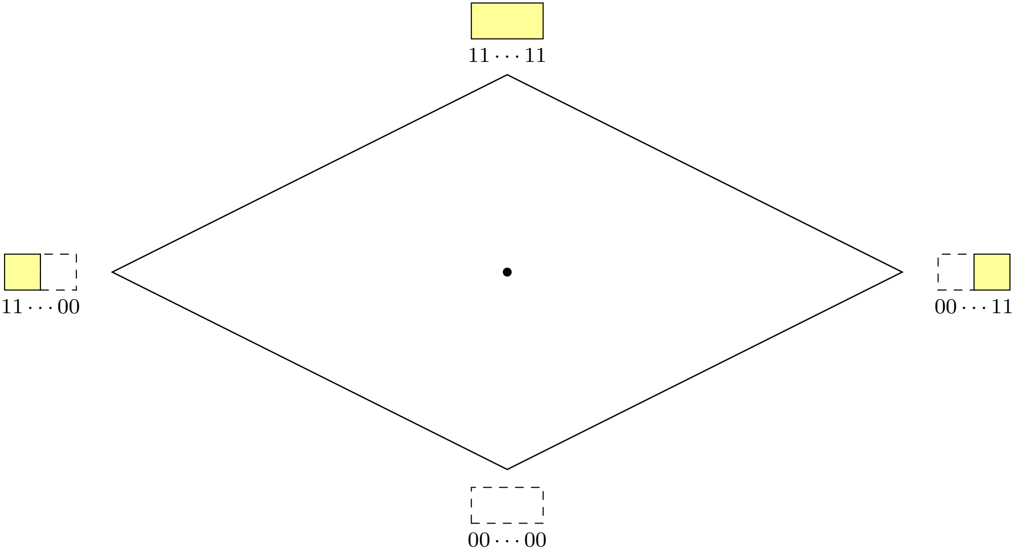
Compression of the Problem



Number of Evaluated Antennas and Computational Time

	plate (8×4)	plate (14×7)	sphere
electrical size (ka)	0.5	0.5	0.5
basis functions (N)	180	567	900
number of iterations (I)	71	279	380
evaluated antennas	**10332**	**119420**	**270129**
realized Q/Q_{lb}	1.57	1.45	1.51
edge removal $(p = \infty)$	0.30 s	23.5 s	79.7 s
edge removal $(p = 50)$	**0.28** s	**19.4**	**63.6** s
edge removal $(p = 1)$	0.43 s	23.3 s	84.9 s
classical pixel removal	**10** s	**1437** s	**10500** s

Moving in the Solution Space, Part #1



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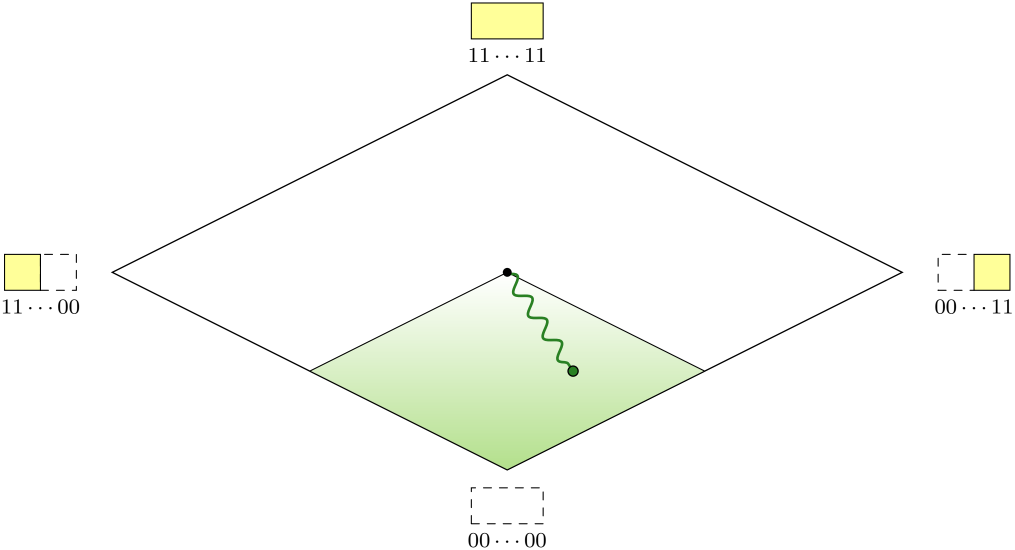
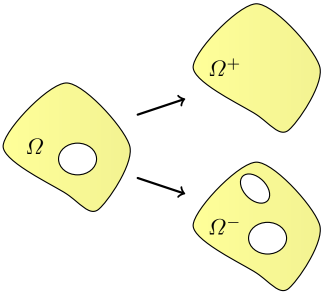


Shape Reconstruction



Adding and removing DOF.

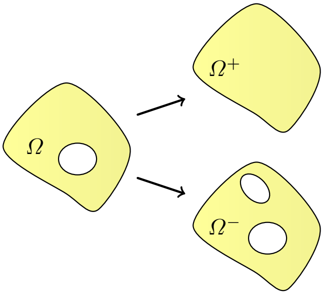
Shape Reconstruction

- Basis functions can be added back (shape reconstruction).

$$[\mathbf{I}_{\mathcal{E}\cup\mathcal{B}}] = \mathbf{C}_{\mathcal{E}\cup b} \begin{bmatrix} \mathbf{y}_f + \dfrac{x_{f1}}{z_1}\mathbf{x}_1 & \cdots & \mathbf{y}_f + \dfrac{x_{fb}}{z_b}\mathbf{x}_b & \cdots \\ -\dfrac{x_{f1}}{z_1} & \cdots & -\dfrac{x_{fb}}{z_b} & \cdots \end{bmatrix} l_f V_0$$

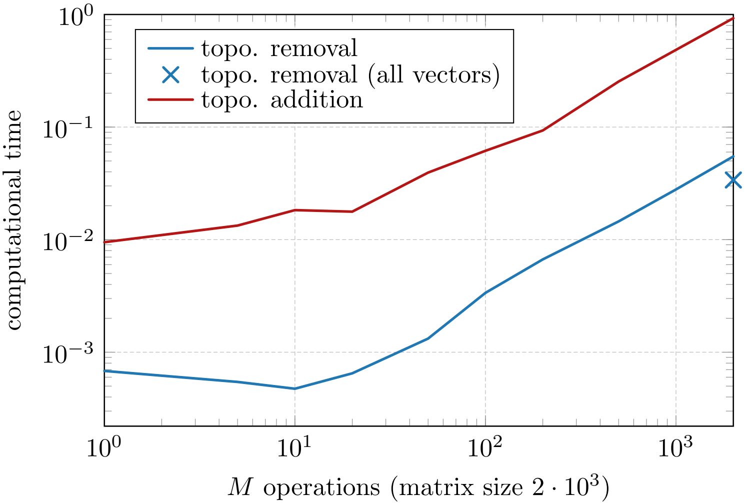
where

$$\mathbf{x}_b = \mathbf{Y}\mathbf{z}_b, \quad z_b = Z_{bb} - \mathbf{z}_b^{\mathrm{T}}\mathbf{x}_b.$$

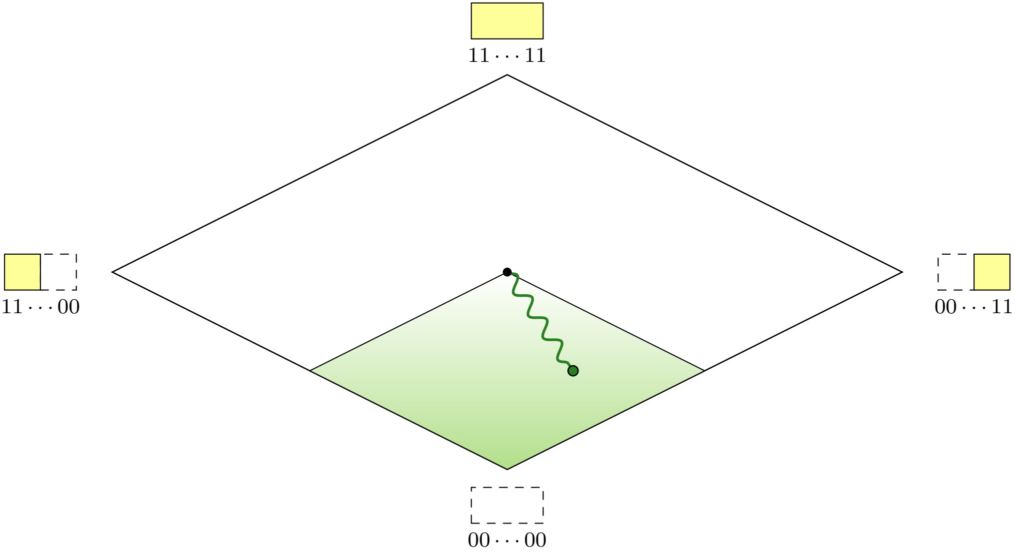


Adding and removing DOF.

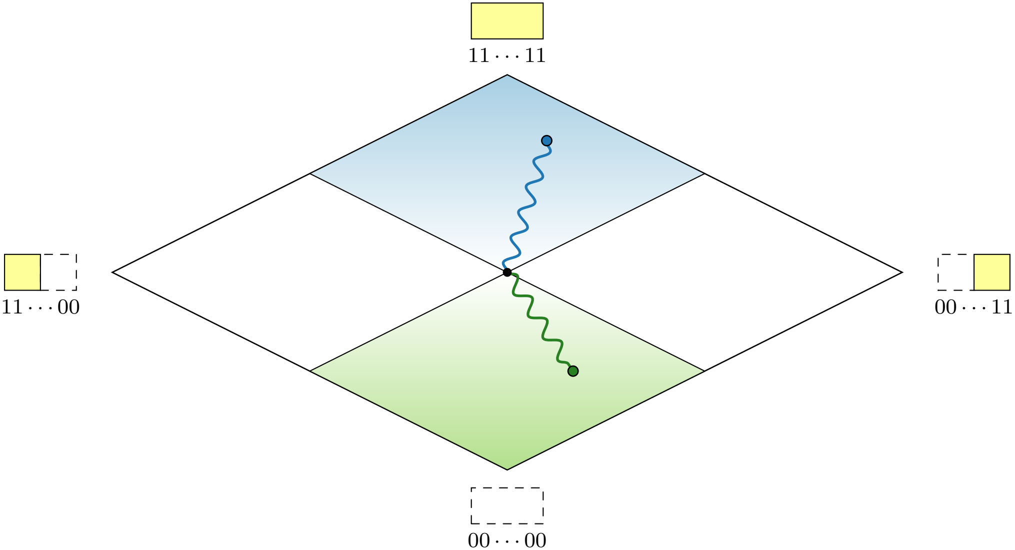
Price to Pay for Reconstruction



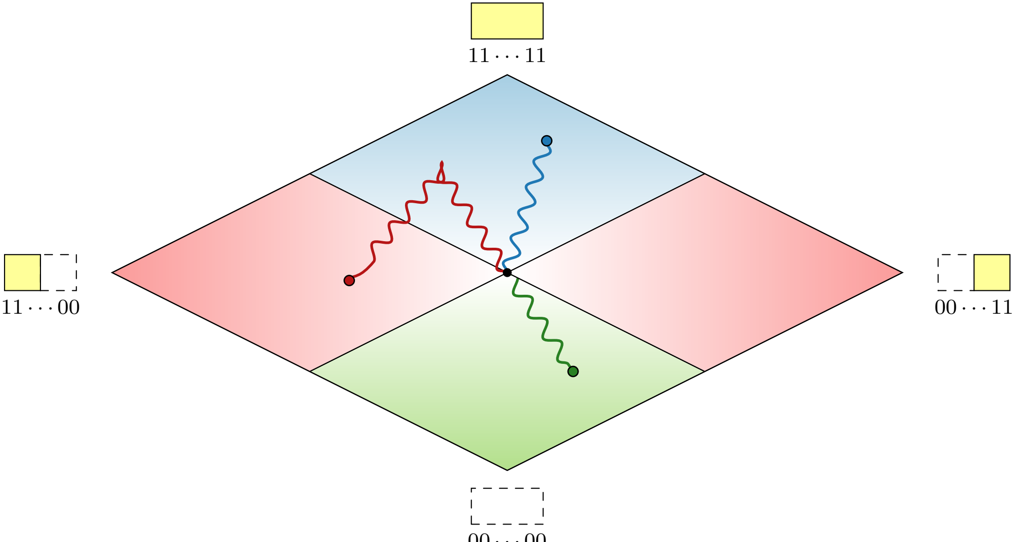
Moving in the Solution Space, Part #2



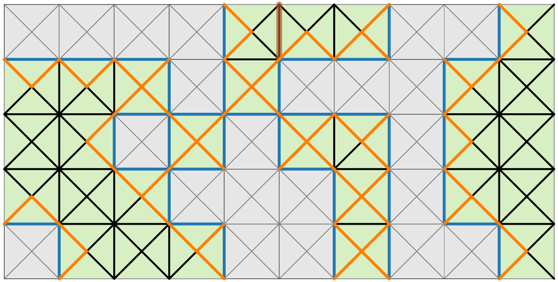
Moving in the Solution Space, Part #2



Moving in the Solution Space, Part #2

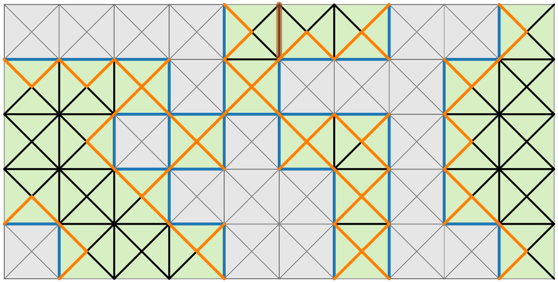


Nearest Neighbor (NN) Search



An initial sample of topology sensitivity investigation.

Nearest Neighbor (NN) Search



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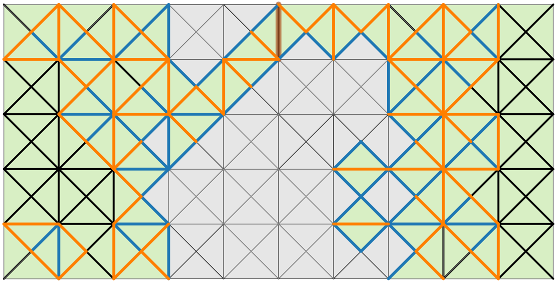
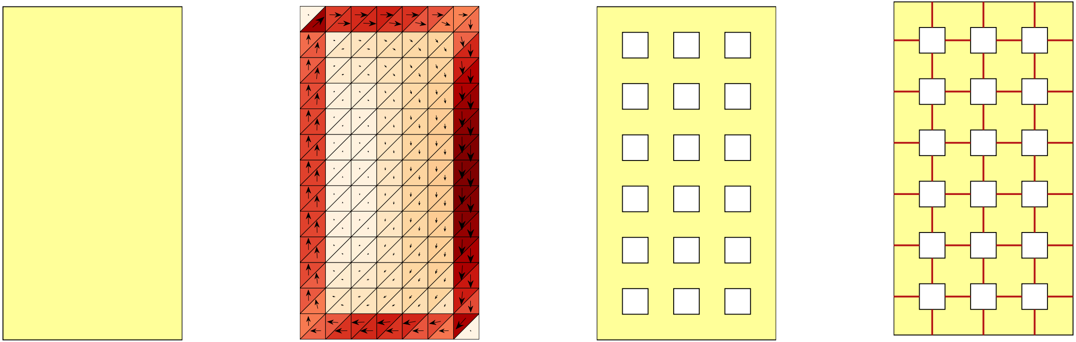


The final sample resulting from a (NN) search.

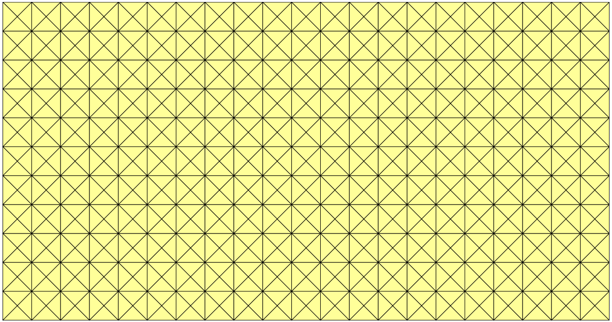
Live demonstration in MATLAB.

All Approaches to Synthesis at Once

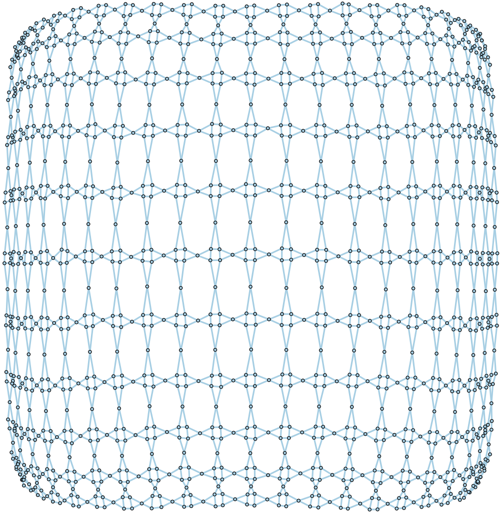
Do not find an approximative solution of the exact model but, instead, find an exact solution of the approximate model.



Reduction of the Complexity



Full grid of 21×11 pixels ($N = 1354$).



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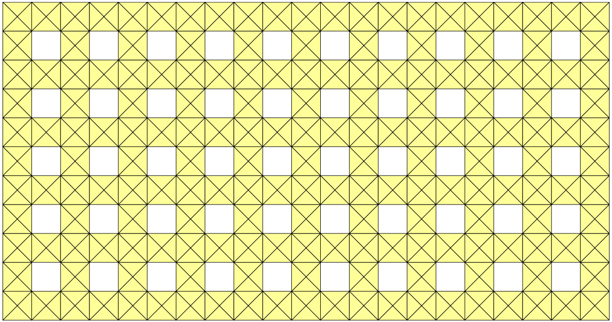
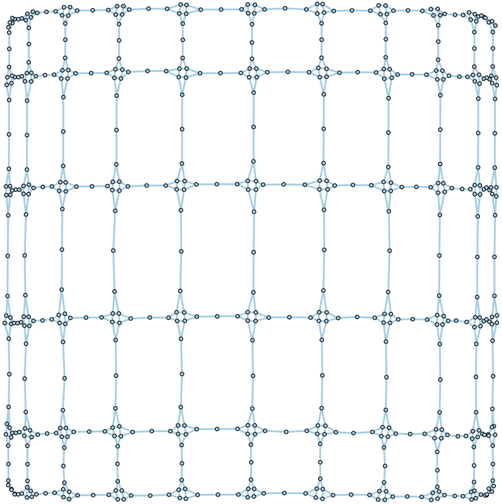
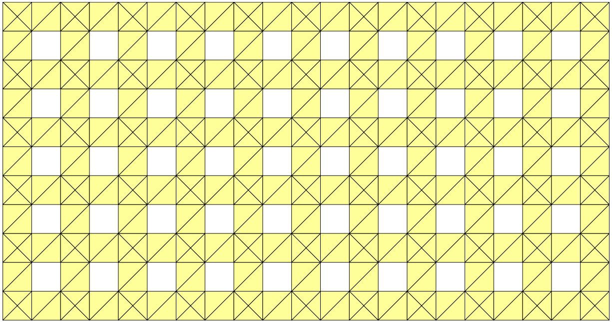


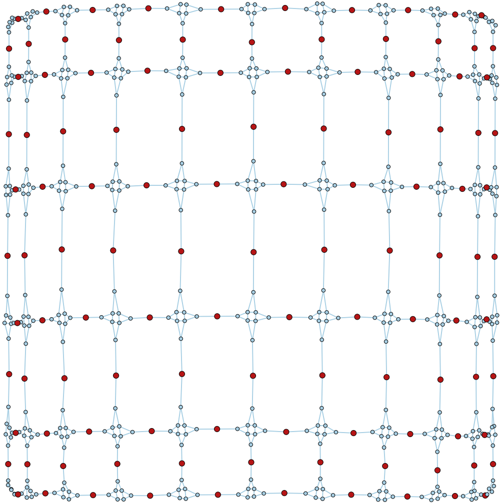
Truncated grid of 21 × 11 pixels ($N = 954$).



Reduction of the Complexity



Truncated grid of 21 × 11 pixels with modified mesh ($N = 115$).



Concluding Remarks

What has been done[7]...

- Inversion-free structure perturbation (removal/addition).
- Evaluation of topology sensitivity, greedy algorithm.
- Vectorization and parallelization friendly algebraic derivation.

[7] M. Capek, L. Jelinek, and M. Gustafsson, "Shape synthesis based on topology sensitivity," , 2018, submitted, arxiv: 1808.02479. [Online]. Available: https://arxiv.org/abs/1808.02479

Concluding Remarks

What has been done[7]...

- Inversion-free structure perturbation (removal/addition).
- Evaluation of topology sensitivity, greedy algorithm.
- Vectorization and parallelization friendly algebraic derivation.

Topics of ongoing research

- Analysis of existing designs – can they be improved?
- Add topology sensitivity into heuristic optimization as a local step.
- Utilization for “data mining” (machine learning).
- Further study of graph representation and formal synthesis problem.
- Admittance matrix pivots (big data, graph clustering).

[7] M. Capek, L. Jelinek, and M. Gustafsson, “Shape synthesis based on topology sensitivity,” , 2018, submitted, arxiv: 1808.02479. [Online]. Available: https://arxiv.org/abs/1808.02479

Questions?

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January 16, 2019
version 1.1
The presentation is available at ▶ capek.elmag.org

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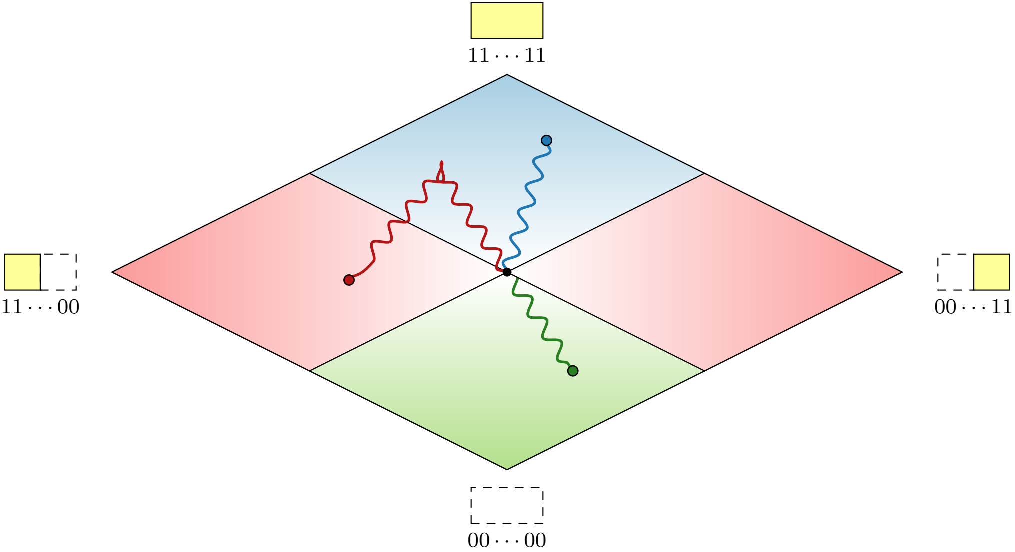
Moving in the Solution Space



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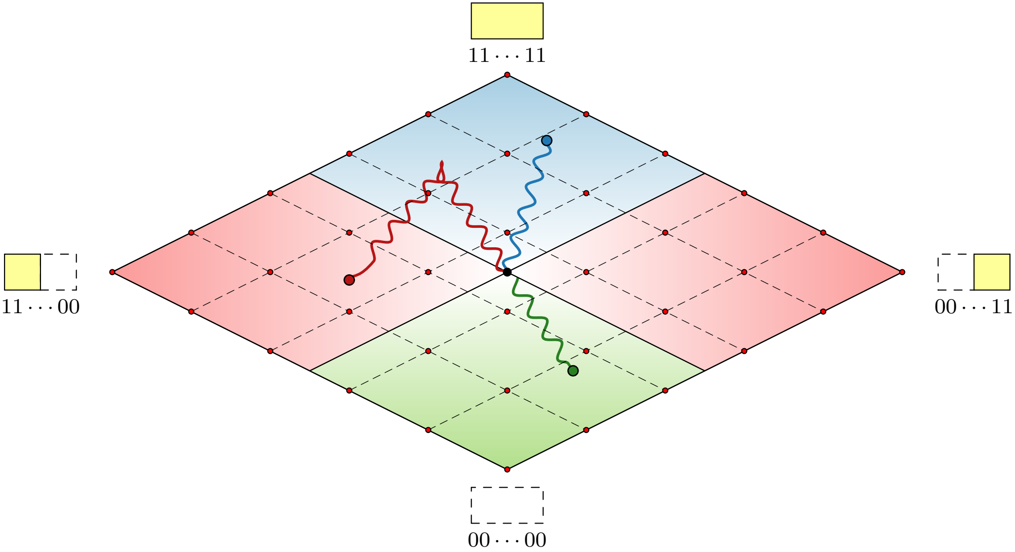

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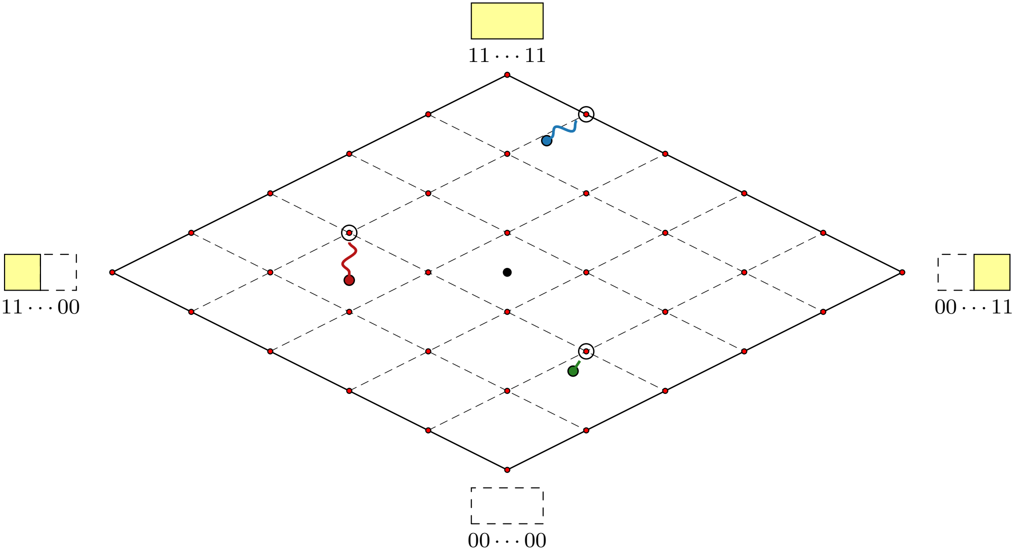

Synthesis – Generalized Framework

Complete and general description of synthesis.

Desired quantity: $\hat{\mathbf{I}}$ (source current), given quantity: $\mathbf{Y}_{\Omega}$ (source region).

$$\hat{\mathbf{I}} = \left(\mathbf{1} - \mathbf{Y}_{\mathbf{G}} \mathbf{C}_{\mathcal{B}} \left(\mathbf{Z}_{\mathrm{L}}^{-1} + \mathbf{C}_{\mathcal{B}}^{\mathrm{T}} \mathbf{Y}_{\mathbf{G}} \mathbf{C}_{\mathcal{B}}\right)^{-1} \mathbf{C}_{\mathcal{B}}^{\mathrm{T}}\right) \mathbf{Y}_{\mathbf{G}} \mathbf{C}_{\mathcal{F}} \boldsymbol{v} V_0$$

$$\hat{\mathbf{I}} = \left(\mathbf{1} - \mathbf{P}\right) \mathbf{Y}_{\Omega} \mathbf{V}$$

- $\mathbf{Y}_{\Omega}$ initial system to be optimized
- $\mathbf{V}$ excitation (external/boundary condition)
- $\mathbf{I}$ solution to original (arbitrarily shaped) structure Ω
- $\mathbf{P}$ (any) modification of the initial (arbitrarily shaped) structure Ω

Computational Complexity

Characterization of the synthesis problem

Number of inner edges	N
Levels of the tree	$N+1$
Total number of solutions	2^N
Number of connections down	$N-n$
Number of connections up	n
Number of nodes at the n-th level	$\dfrac{N!}{n!\,(N-n)!} = \dbinom{N}{n}$
Number of connections down from the n-th level	$n\dbinom{N}{n}$