Inversion-Free Evaluation of Small Geometry Perturbation in Method of Moments

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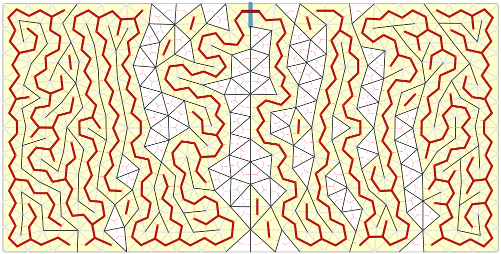
Outline

1. Problem Parametrization
2. Inversion-free Solution of Linear System
3. Graph Representation
4. Monte Carlo Analysis (Q-factor Optimization)
5. Heuristically Restarted Topology Sensitivity
6. Concluding Remarks



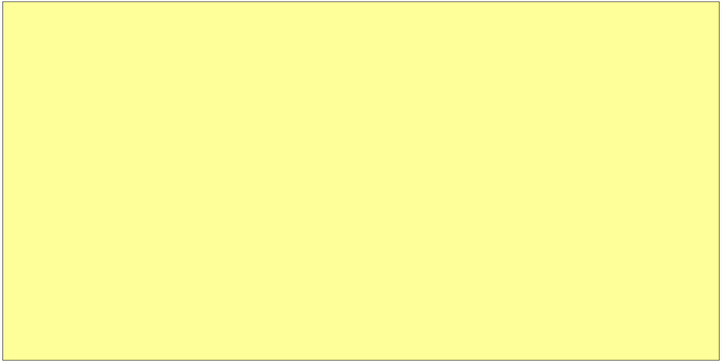
(Sub-)optimal solution of Q-factor minimization over triangularized grid, 753 DOF.

This talk concerns:

- electric currents in vacuum,
- time-harmonic quantities, *i.e.*, $\boldsymbol{\mathcal{A}}(\boldsymbol{r}, t) = \mathrm{Re}\{\boldsymbol{A}(\boldsymbol{r}) \exp(\mathrm{j}\omega t)\}$.

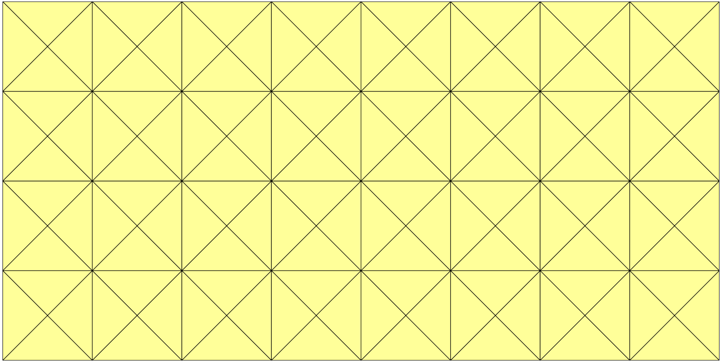
Degrees of Freedom

Ω



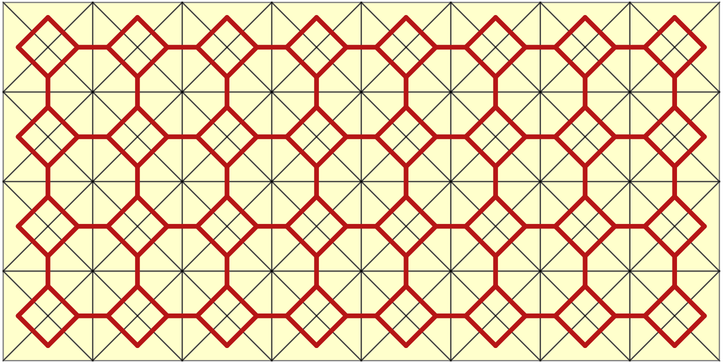
Degrees of Freedom

$$\Omega \to \{T_t\}$$



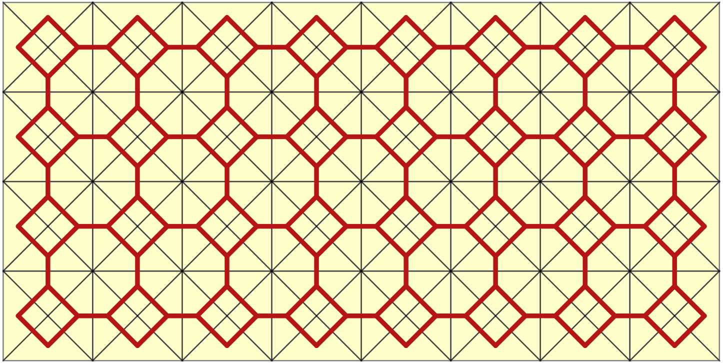
Degrees of Freedom

$$\Omega \rightarrow \{T_t\} \rightarrow \{\boldsymbol{\psi}_n(\boldsymbol{r})\}$$



Degrees of Freedom

$$\Omega \to \{T_t\} \to \{\boldsymbol{\psi}_n(\boldsymbol{r})\} \to \boldsymbol{g}$$



- $\mathbf{g} \in \{0,1\}^{N \times 1}$ is characteristic vector (discretized characteristic function)

Shape Optimization

Capability to effectively remove/add a degree of freedom.[1]

[1] M. Capek, L. Jelinek, and M. Gustafsson, “Shape synthesis based on topology sensitivity,” *IEEE Trans. Antennas Propag.*, vol. 67, no. 6, pp. 3889 –3901, 2019. DOI: 10.1109/TAP.2019.2902749

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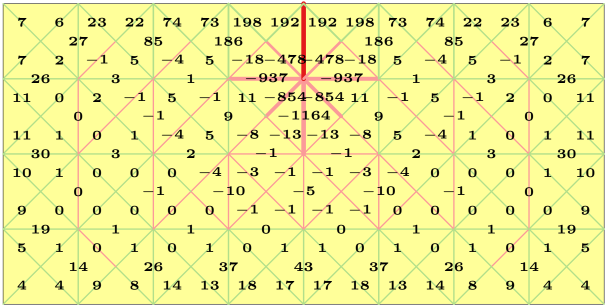
- Perfectly compatible with method of moments;
  - basis functions used as DOF.



Example of topology sensitivity, $ka = 1/2$, plate fed in the middle.

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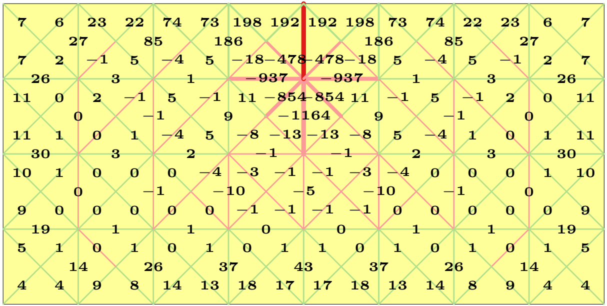
Shape Optimization

Capability to effectively remove/add a degree of freedom.[1]

- Perfectly compatible with method of moments;
  - basis functions used as DOF.
- Inversion-free for the smallest perturbations;
  - gradient-based shape optimization possible deterministically.



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Removing and Adding DOF[2]

DOF removal:

$$\widehat{\mathbf{I}} = \left(\mathbf{y}_f - \frac{Y_{fb}}{Y_{bb}}\mathbf{y}_b\right) l_f V_0,$$

Admittance matrix update:

$$\widehat{\mathbf{Y}} = \mathbf{C}^{\mathrm{T}} \left(\mathbf{Y} - \frac{1}{Y_{bb}}\mathbf{y}_b\mathbf{y}_b^{\mathrm{T}}\right)\mathbf{C},$$

DOF addition:

$$\widehat{\mathbf{I}} = \mathbf{C}^{\mathrm{T}} \left(\begin{bmatrix} \mathbf{y}_f \\ 0 \end{bmatrix} + \frac{x_{fb}}{z_b}\begin{bmatrix} \mathbf{x}_b \\ -1 \end{bmatrix}\right) l_f V_0,$$

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$$C_{nn} = \begin{cases} 0 & \Leftrightarrow \quad g_n = b \\ 1 & \Leftrightarrow \quad \text{otherwise} \end{cases}$$

$$\mathbf{x}_b = \mathbf{Y}\widetilde{\mathbf{z}}_b, \quad z_b = \widetilde{Z}_{bb} - \widetilde{\mathbf{z}}_b^{\mathrm{T}}\mathbf{x}_b$$

$$C_{mn} = \begin{cases} 1 & \Leftrightarrow \quad g_n = S(m) \\ 0 & \Leftrightarrow \quad \text{otherwise} \end{cases}$$

- All columns of **C** matrix containing solely zeros are eliminated before use.

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Topology Sensitivity

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$$\tau\left(\mathcal{P}, \mathit{\Omega}\right) = -\left(\mathcal{P}\left(\mathbf{I}\right) - \mathcal{P}\left(\widehat{\mathbf{I}}\right)\right)$$

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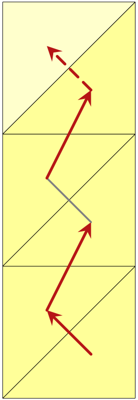
For example, Q-factor is evaluated as

$$\mathcal{P}\left(\mathbf{I}\right) \equiv Q = \frac{\mathbf{I}^{\mathrm{H}}\mathbf{W}\mathbf{I} + \left|\mathbf{I}^{\mathrm{H}}\mathbf{X}\mathbf{I}\right|}{\mathbf{I}^{\mathrm{H}}\mathbf{R}\mathbf{I}},$$

$\mathbf{W} = \omega\partial\mathbf{X}/\partial\omega$, $\mathbf{Z} = \mathbf{R} + \mathrm{j}\mathbf{X} \in \mathbb{C}^{N\times N}$.

Optimization variable: binary vector $\mathbf{g}$;

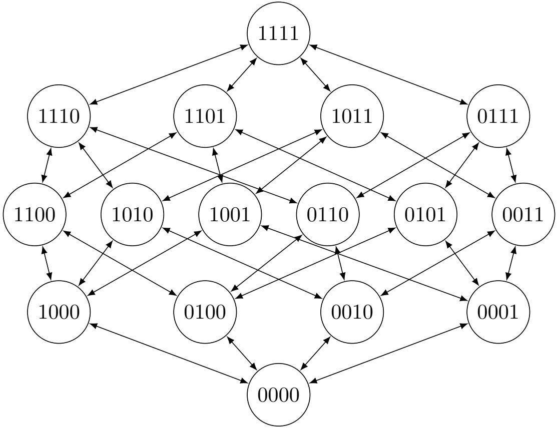
- analogy to characteristic function,
- determines which DOF are enabled/disabled.



Solid – enabled,
dashed – disabled,
grayed – fed edge.

$$\mathbf{g} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Nearest Neighbors: Hamming graph $H(N,2)$ of Vectors $\mathbf{g}$



Solution space for $N = 4$ represented as a hierarchic Hamming graph with nearest neighbors highlighted.

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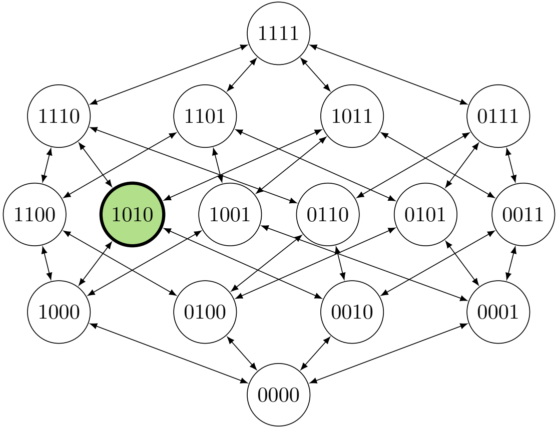

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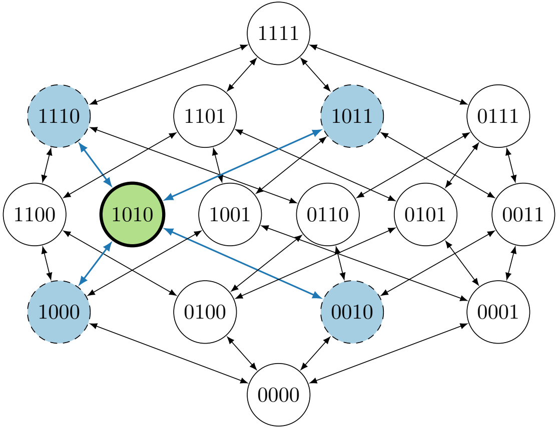

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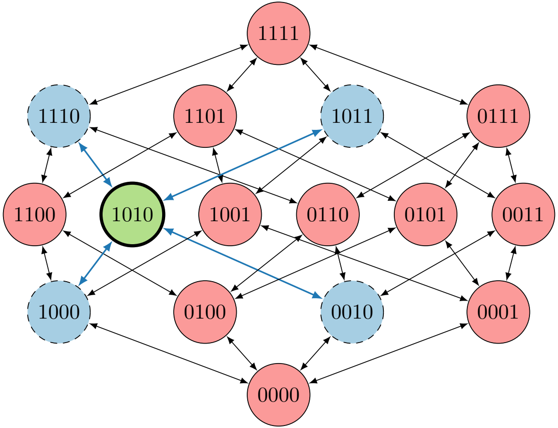

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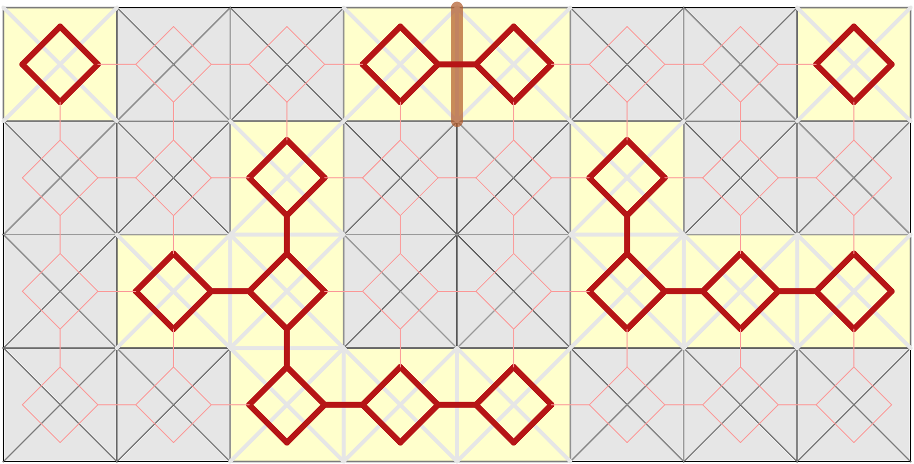
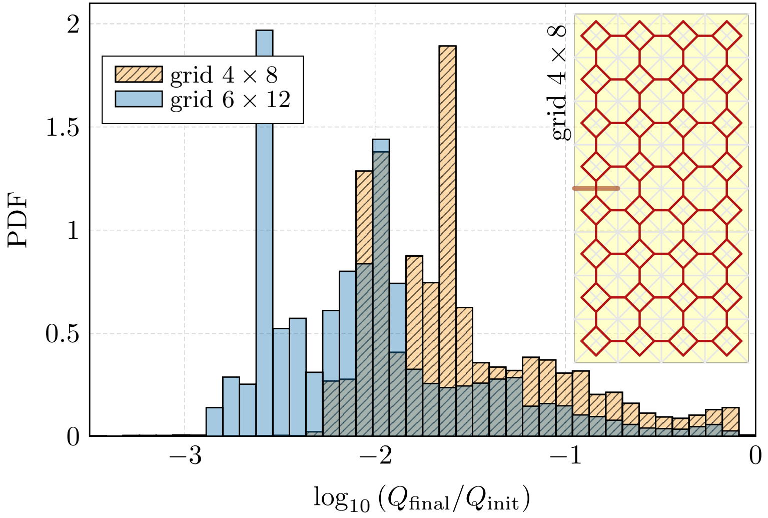


$$i = 0,\ Q/Q_{\mathrm{lb}}^{\mathrm{TM}} = 119.35$$

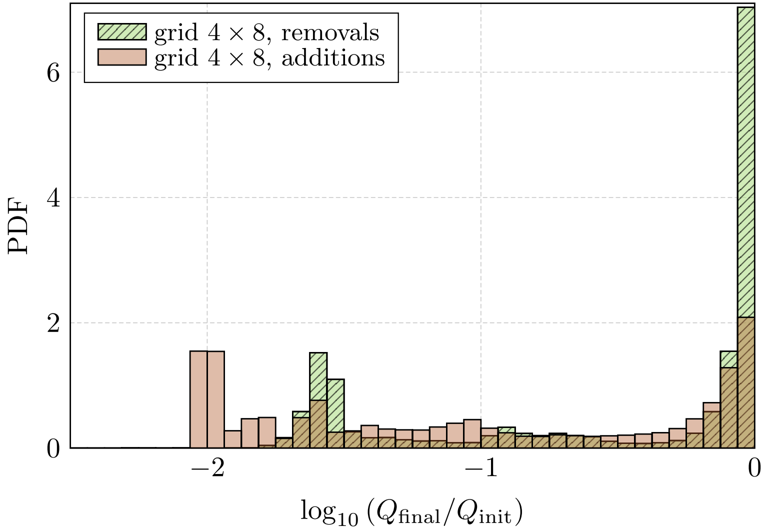
Shape Optimization: Monte Carlo Analysis

- Topology sensitivity over nearest neighbors, fitness function: Q-factor.
- Starting seeds $\boldsymbol{g}$ selected randomly, updated till local minimum reached.
- Number of restarts $N = 5 \cdot 10^4$, *i.e.*, statistics doable...

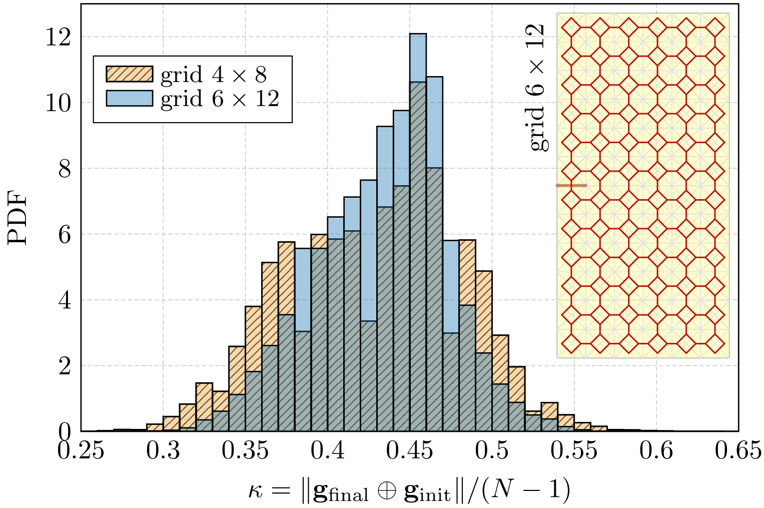
Relative Improvement of Q-factor (PDF)



Effect of Removals × Additions Only (PDF)



Relative Number of Required Improvements (PDF)



Performance of Found Structures (CDF, PDF)

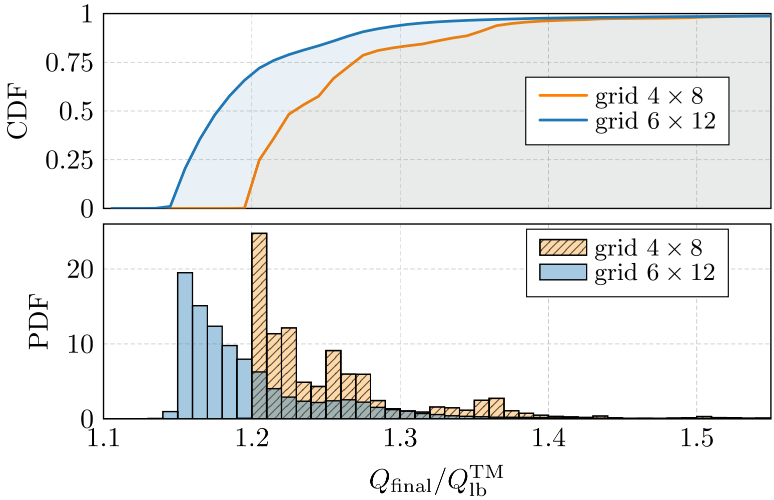


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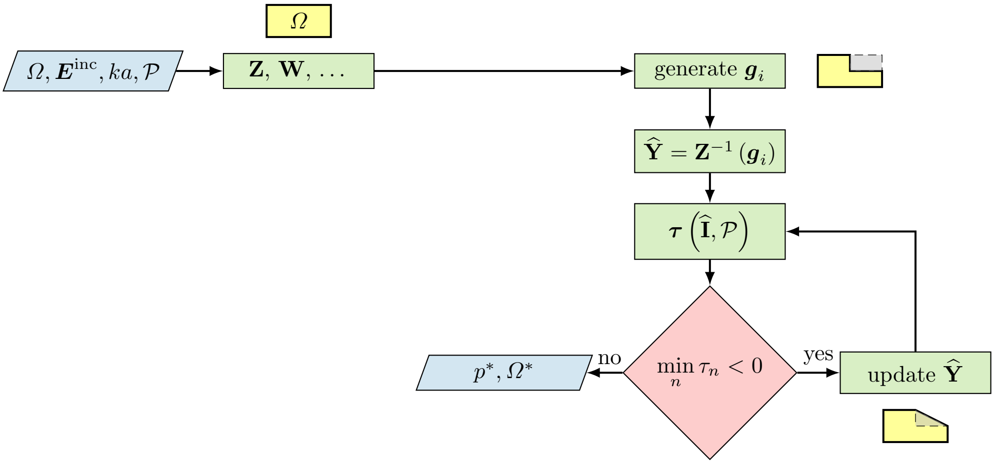
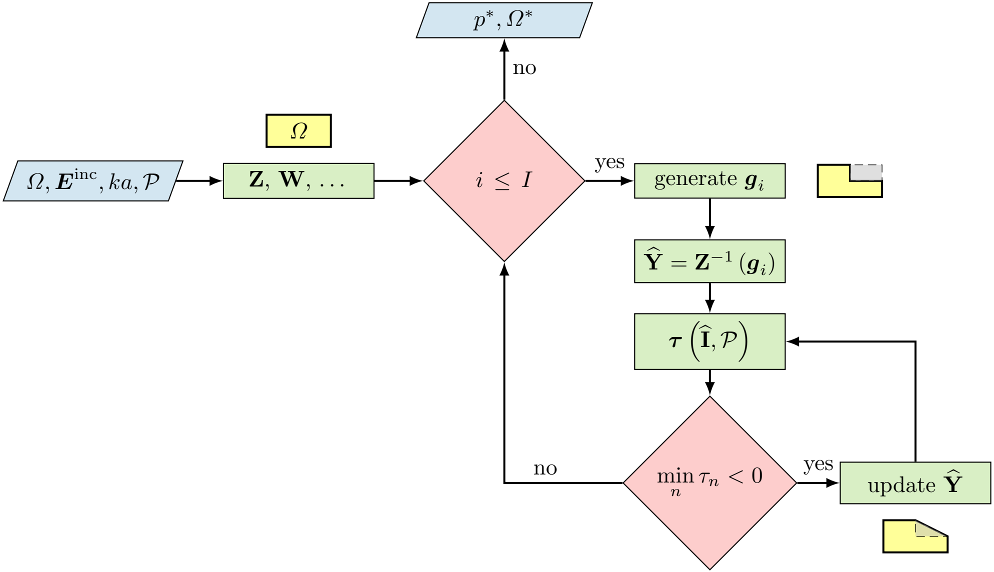
plate	4×8	6×12	8×16
DOF, N	180	414	744
runs, I	$5 \cdot 10^4$	$5 \cdot 10^4$	$1 \cdot 10^3$
comp. time, T [s]	$2.4 \cdot 10^3$	$5.8 \cdot 10^4$	$1.2 \cdot 10^4$
evaluated shapes	$7.2 \cdot 10^8$	$3.9 \cdot 10^9$	$2.6 \cdot 10^8$
shapes per second	$3 \cdot 10^5$	$7 \cdot 10^4$	$2 \cdot 10^4$
comp. time per run, T/I [s]	$4.8 \cdot 10^{-2}$	$1.2 \cdot 10^0$	$1.2 \cdot 10^1$
evaluated shapes per run	$1.4 \cdot 10^4$	$7.8 \cdot 10^4$	$2.6 \cdot 10^5$
$Q_{\mathrm{min}}/Q_{\mathrm{lb}}^{\mathrm{TM}}$	1.18	1.12	1.11

Computer: CPU Threadripper 1950X (3.4 GHz), 128 GB RAM.


Ω
$\Omega, \boldsymbol{E}^{\mathrm{inc}}, ka, \mathcal{P}$
$\mathbf{Z}, \mathbf{W}, \ldots$
generate $\boldsymbol{g}_i$
$\widehat{\mathbf{Y}} = \mathbf{Z}^{-1}(\boldsymbol{g}_i)$
$\boldsymbol{\tau}\left(\widehat{\mathbf{I}}, \mathcal{P}\right)$
$\min_n \tau_n < 0$
no
yes
p^*, Ω^*
update $\widehat{\mathbf{Y}}$


p^*, Ω^*
no
Ω
$\Omega, \boldsymbol{E}^{\mathrm{inc}}, ka, \mathcal{P}$
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$i \leq I$
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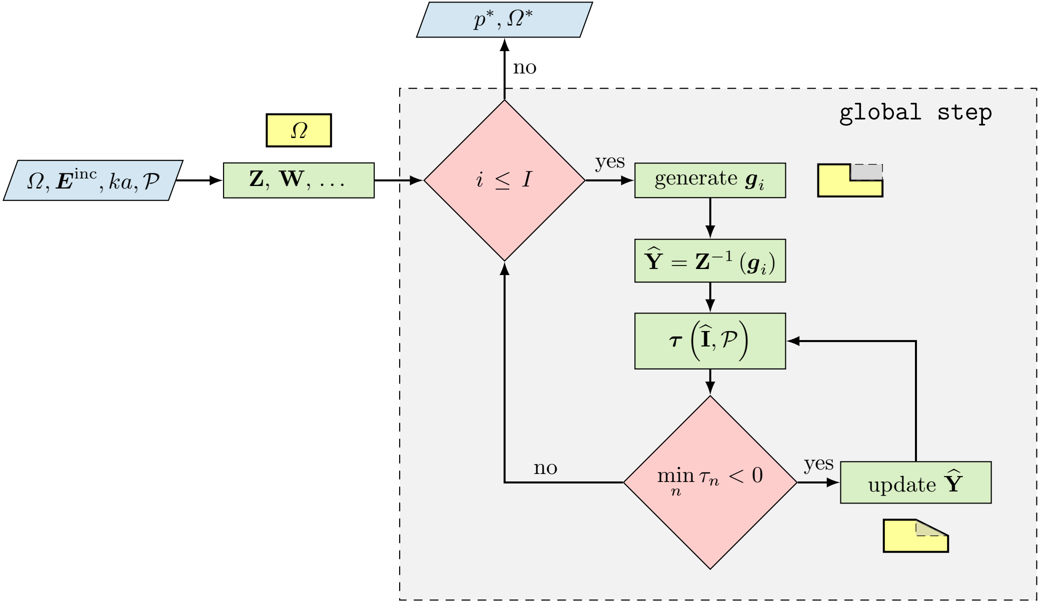
p^*, Ω^*
no
global step
Ω
$\Omega, \boldsymbol{E}^{\mathrm{inc}}, ka, \mathcal{P}$
$\mathbf{Z}$, $\mathbf{W}$, ...
$i \leq I$
yes
generate $\boldsymbol{g}_i$
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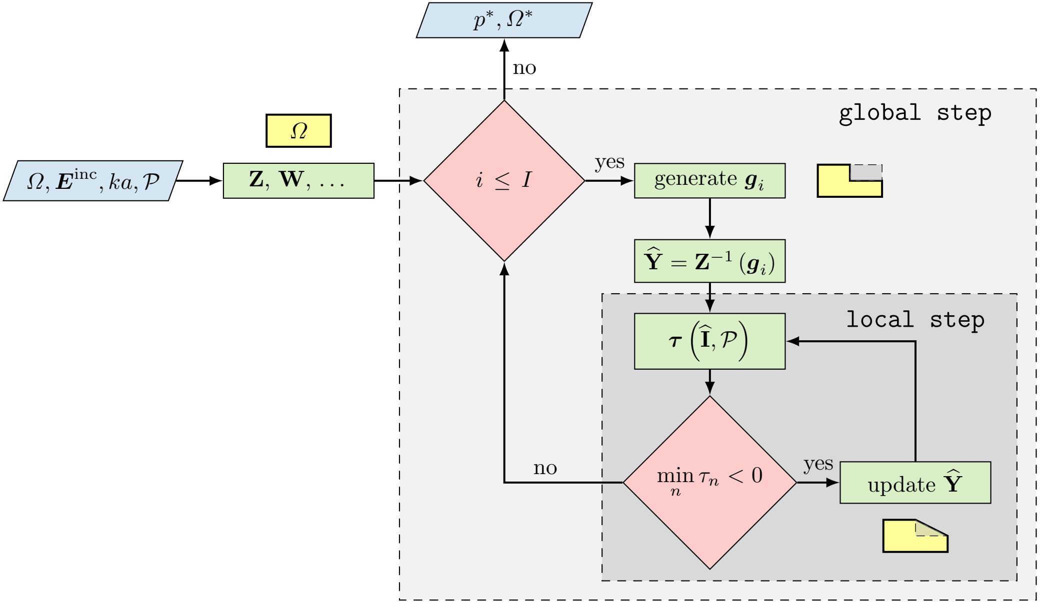
$p^*, \varOmega^*$
no
global step
$\varOmega$
$\varOmega, \boldsymbol{E}^{\mathrm{inc}}, ka, \mathcal{P}$
$\mathbf{Z}, \mathbf{W}, \dots$
$i \leq I$
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Topology Sensitivity (TS) & Heuristic Algorithm (HA)

TS Local, gradient-based, very fast.

HA Robust, able to restart TS.

TS & HA Moves only through local minima of an optimization problem!

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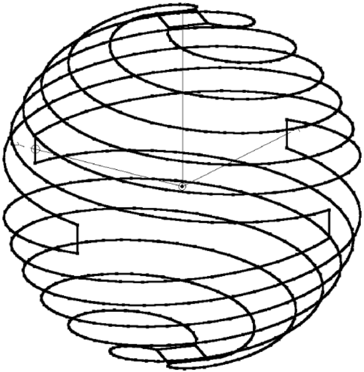
- **TS & HA** Moves only through local minima of an optimization problem!



Four-arm folded helix[3].

[0]S. R. Best, "Low Q electrically small linear and elliptical polarized spherical dipole antennas," *IEEE Trans. Antennas Propag.*, vol. 53, no. 3, pp. 1047–1053, 2005. DOI: 10.1109/TAP.2004.842600

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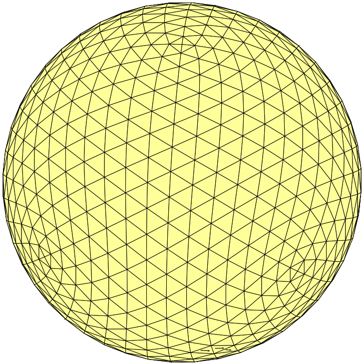
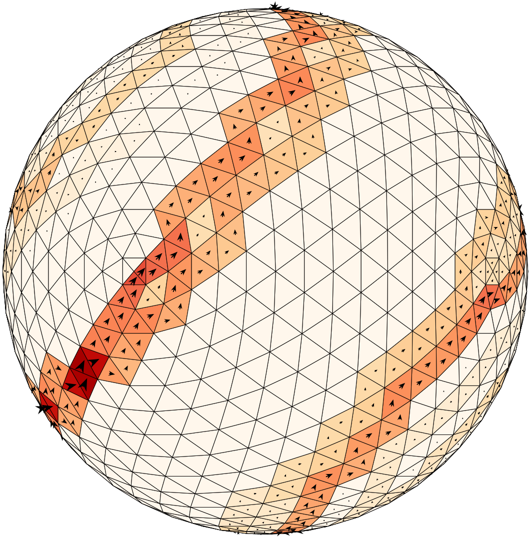


Discretized spherical shell (1536 triangles, 2304 DOF).

TS + HA (SOGA) optimization:

Electrical size	$ka = 0.2$
Triangles	1536
DOF	2304
Agents	224
Iterations	500
Evaluated antennas	$4.6 \cdot 10^{9}$
Size of solution space	$2^{2303} \approx 1.63 \cdot 10^{91}$
Computational time	205 hours
$Q/Q_{\mathrm{lb}}^{\mathrm{TM}}$	0.826
$Q/Q_{\mathrm{lb}}^{\mathrm{TM+TE}}$	1.205



Computer: CPU Threadripper 1950X (3.4 GHz), 16 cores, 128 GB RAM.

What has been done

- Inversion-free topology sensitivity derived[3].
  - Shape perturbation possible with $\mathcal{O}(N)$, shape update only with outer product.
- Shape optimization possible via effective topology optimization[4].
  - Monte Carlo analysis based on greedy step through nearest neighbors.
  - Termination criteria from fundamental bounds.

[3] M. Capek, L. Jelinek, and M. Gustafsson, "Shape synthesis based on topology sensitivity," *IEEE Trans. Antennas Propag.*, vol. 67, no. 6, pp. 3889 –3901, 2019. DOI: 10.1109/TAP.2019.2902749

[4] M. Capek, L. Jelinek, and M. Gustafsson, "Inversion-free evaluation of nearest neighbors in method of moments," , 2019, In press (AWPL). DOI: 10.1109/LAWP.2019.2912459

What has been done

- Inversion-free topology sensitivity derived[3].
  - Shape perturbation possible with $\mathcal{O}(N)$, shape update only with outer product.
- Shape optimization possible via effective topology optimization[4].
  - Monte Carlo analysis based on greedy step through nearest neighbors.
  - Termination criteria from fundamental bounds.

Some ideas of ongoing research

- Mature cooperation of heuristics and topology sensitivity.
- Neural networks/machine learning techniques.
- Increase graph coverage;
  - massive parallelization, model surrogation, graph simplification.
- Multi-objective optimization.

[3] M. Capek, L. Jelinek, and M. Gustafsson, "Shape synthesis based on topology sensitivity," *IEEE Trans. Antennas Propag.*, vol. 67, no. 6, pp. 3889 –3901, 2019. DOI: 10.1109/TAP.2019.2902749

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Questions?

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version 1.0
The presentation is available at ▶ capek.elmag.org

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Machine Learning With Topology Sensitivity Procedure

Conventional approach = impedance matrix inversion + fitness function evaluation.

$$\{\varOmega, \boldsymbol{E}^{\mathrm{i}}\} \longrightarrow \underbrace{\boxed{\boldsymbol{J} = \mathcal{L}^{-1}\left(\boldsymbol{E}^{\mathrm{i}}\right)}}_{\mathcal{O}\left(N^3\right)} \xrightarrow{\ \boldsymbol{J}\ } \underbrace{\boxed{p = \mathcal{P}\left(\boldsymbol{J}\right)}}_{\mathcal{O}\left(N\right),\ \mathcal{O}\left(N^2\right)} \longrightarrow p$$

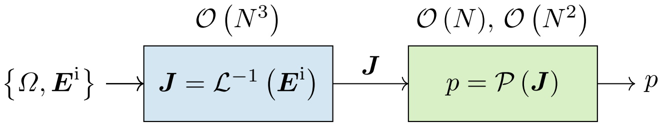
Topology sensitivity:

$$\{\boldsymbol{g}_i\} \longrightarrow \underbrace{\boxed{\text{topo. sens.}}}_{\mathcal{O}\left(N\right),\ \mathcal{O}\left(N^2\right)} \longrightarrow \{p_i\}$$

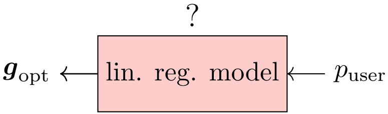
1. Algorithmic complexity reduction.
2. Local optimization algorithm.
3. Excellent for data mining...

Machine Learning With Topology Sensitivity Procedure

Conventional approach = impedance matrix inversion + fitness function evaluation.



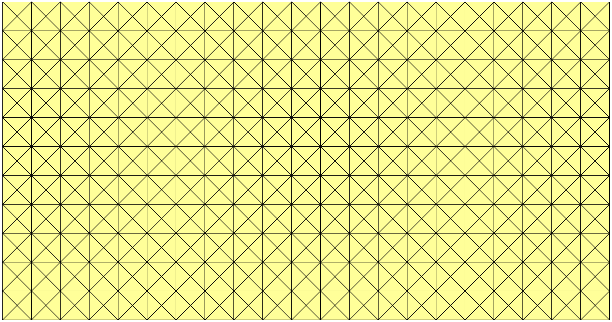
Learned model:



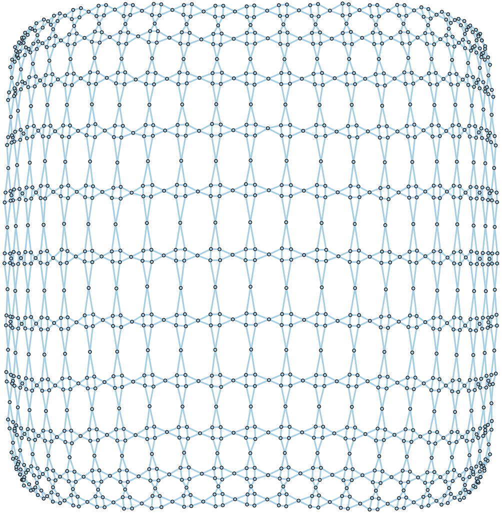
dof, N	180	414	744
shapes per second	$3 \cdot 10^5$	$7 \cdot 10^4$	$2 \cdot 10^4$

Computer: CPU Threadripper 1950X (3.4 GHz), 128 GB RAM.

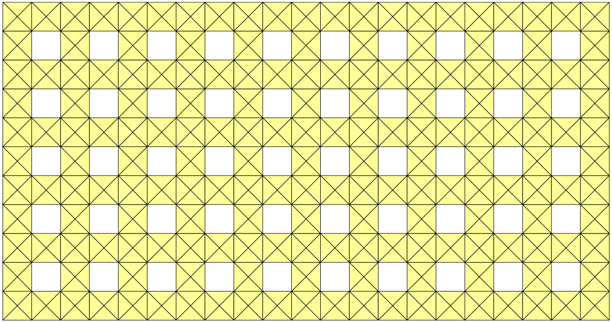
Reduction of Graph (Computational) Complexity



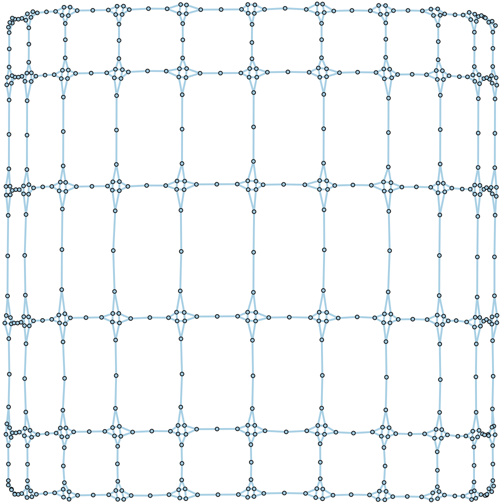
Full grid of 21×11 pixels ($N = 1354$).



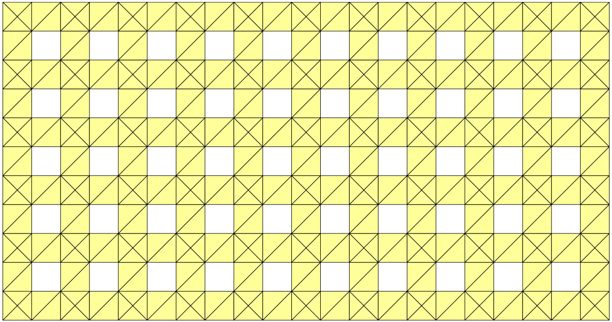
Reduction of Graph (Computational) Complexity



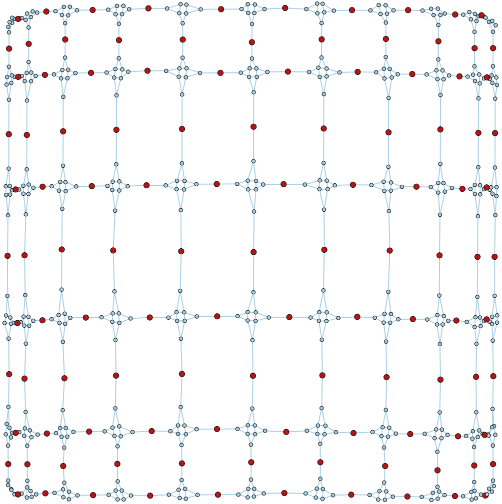
Truncated grid of 21 × 11 pixels ($N = 954$).



Reduction of Graph (Computational) Complexity

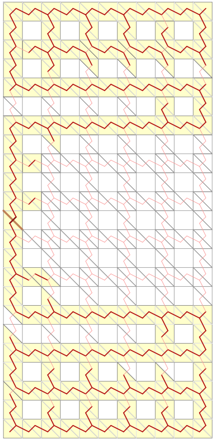


Truncated grid of 21 × 11 pixels with modified mesh ($N = 115$).

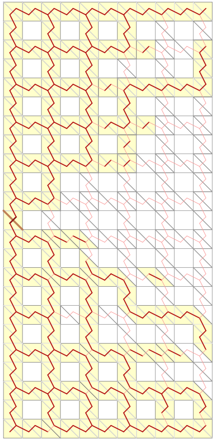


Surrogated Discretized Models (1484 → 450 DOFS), TS+GA

Rectangular plate 2 : 1, fed in the middle, $ka = 1/2$.

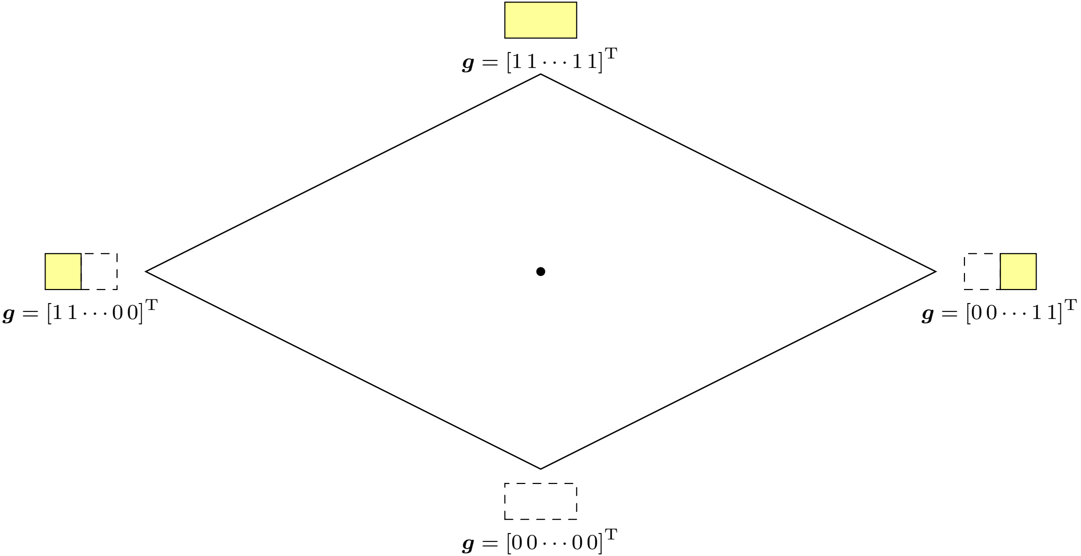


Minimum Q-factor for surrogated model.

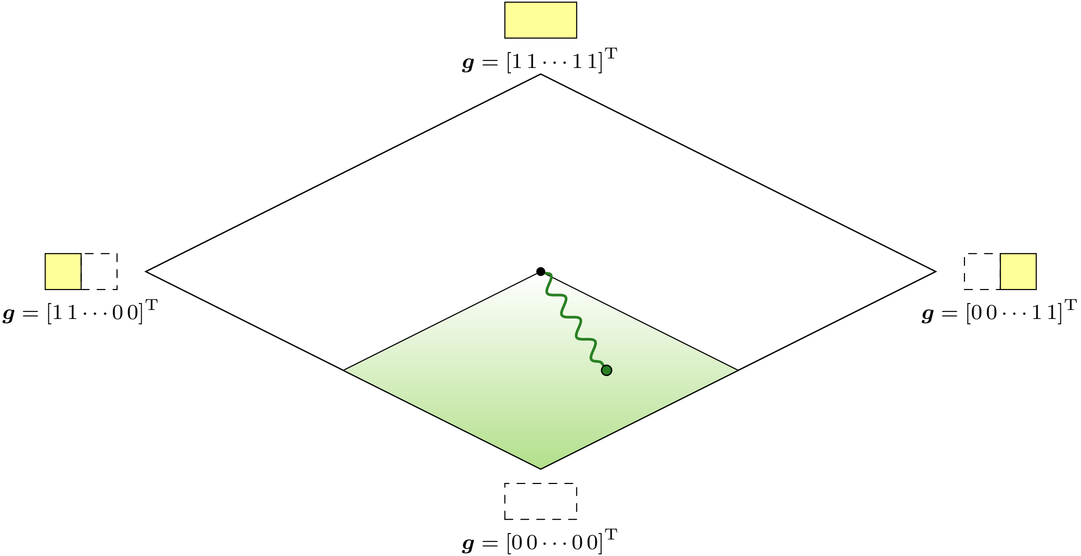


Maximum radiation efficiency.

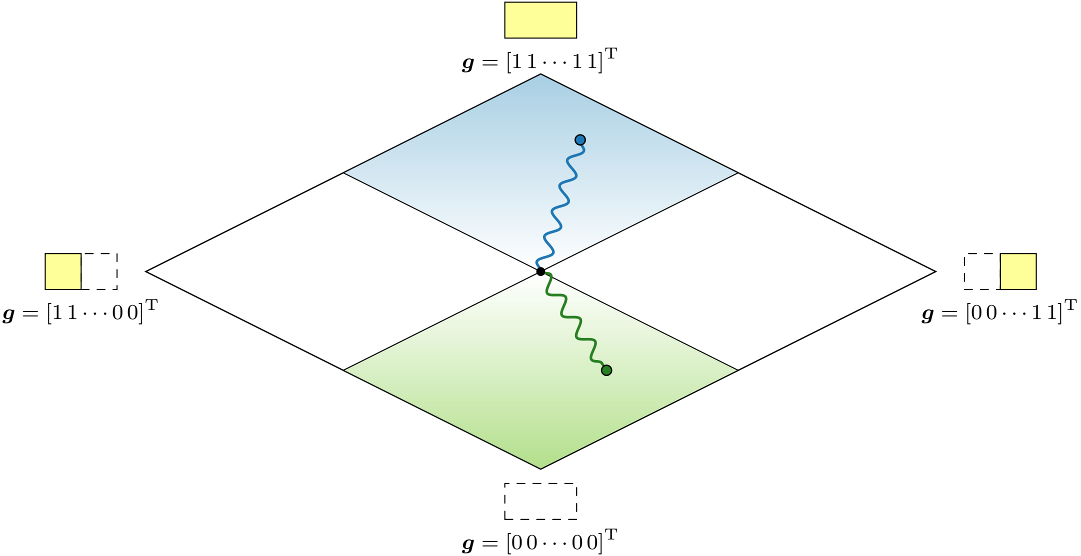
Moving in the Solution Space



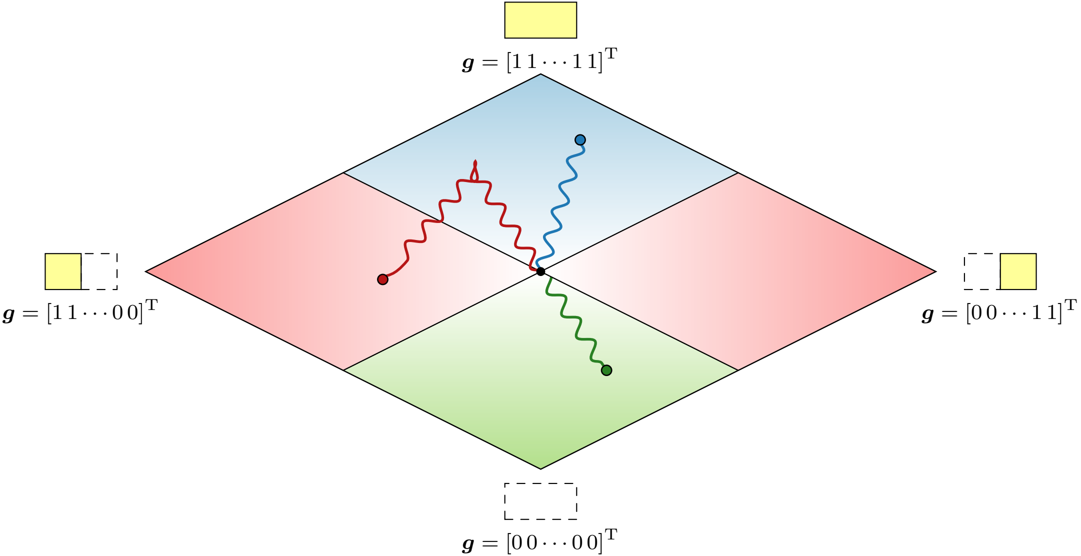
Moving in the Solution Space



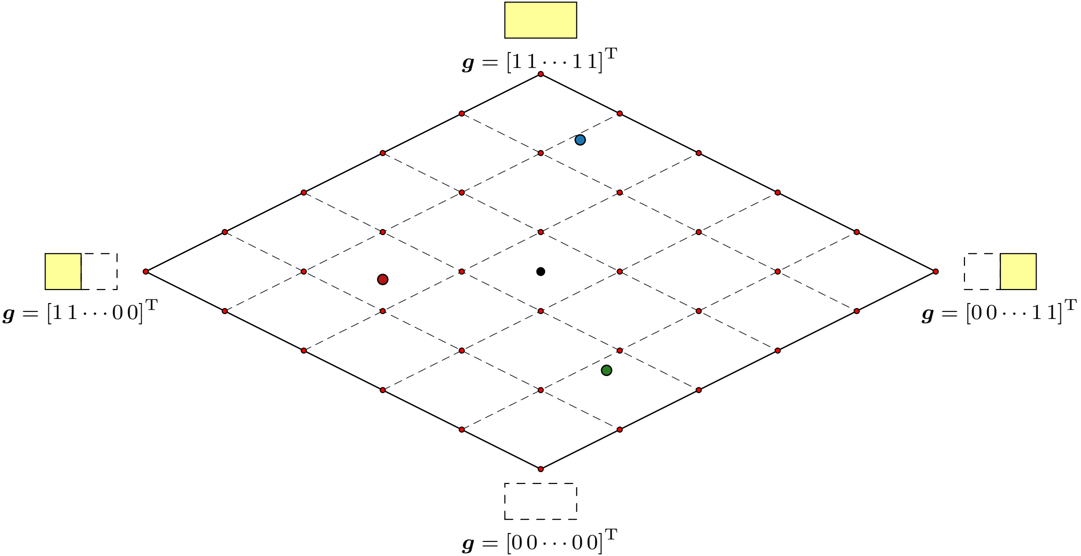
Moving in the Solution Space



Moving in the Solution Space



Moving in the Solution Space



Moving in the Solution Space